Introduction to Neutron Reflectometry (NR)

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Outline

- Principles of Neutron Reflectometry
- Reflectometry of Non-Polarized and Polarized Neutrons from Solid Interfaces
- Neutron Reflectometry from Interfaces with Liquids
- Experimental Aspects of Neutron Reflectometry

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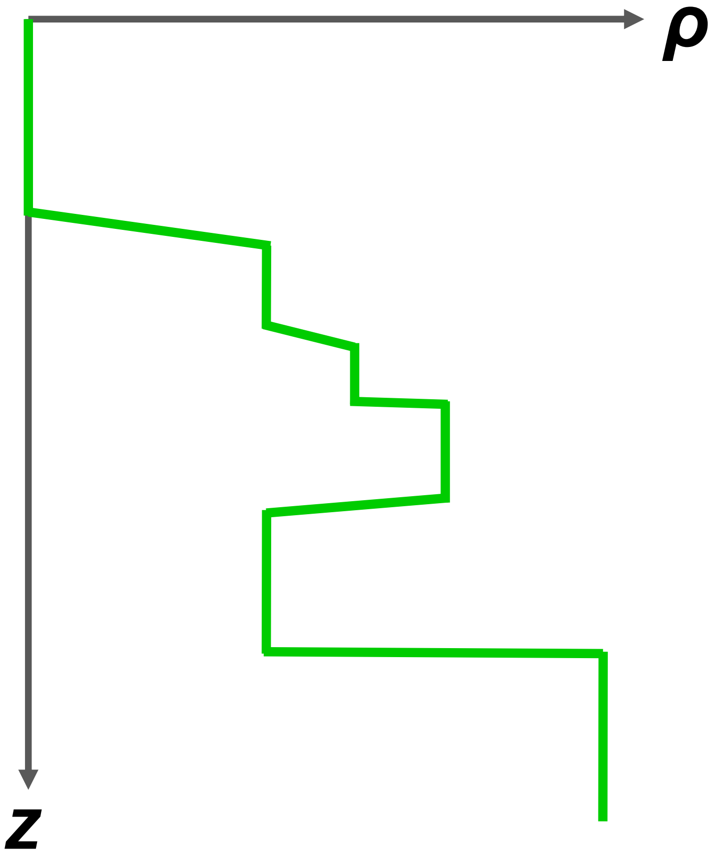
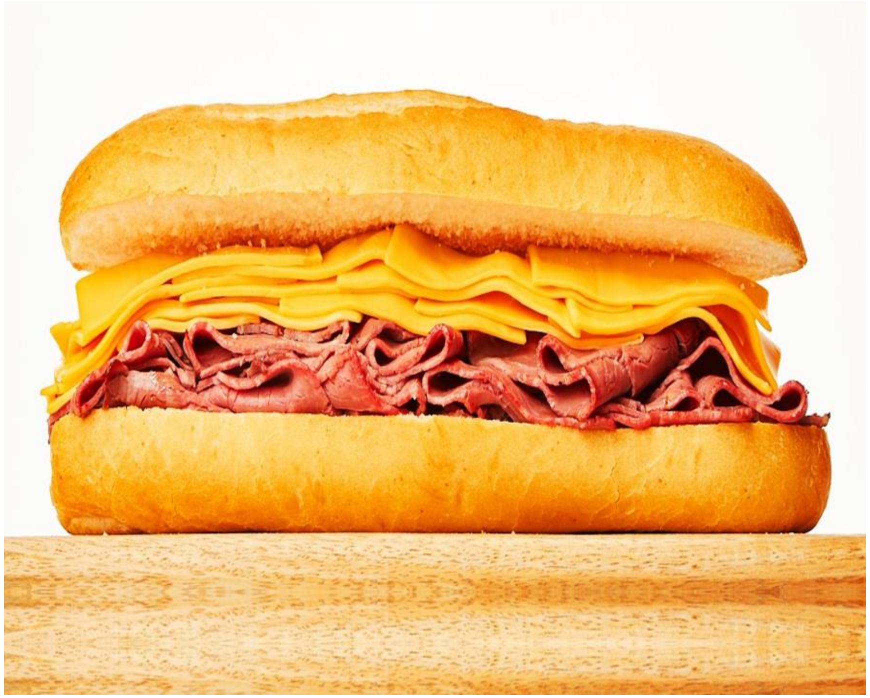
Research Objects

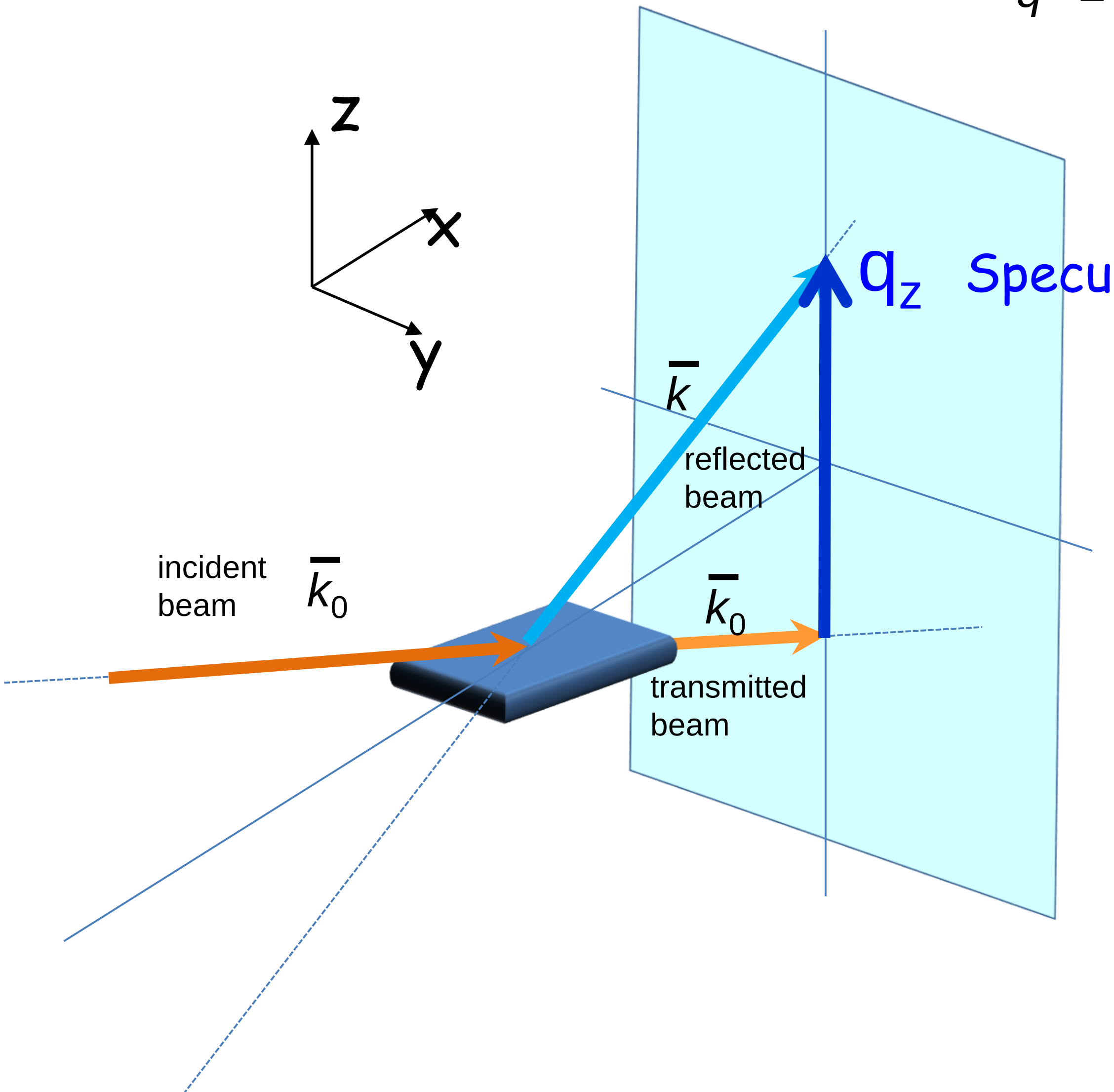
sandwich-like structures





NR modes

$$\bar{q} = \bar{k} - \bar{k}_0$$



NR modes

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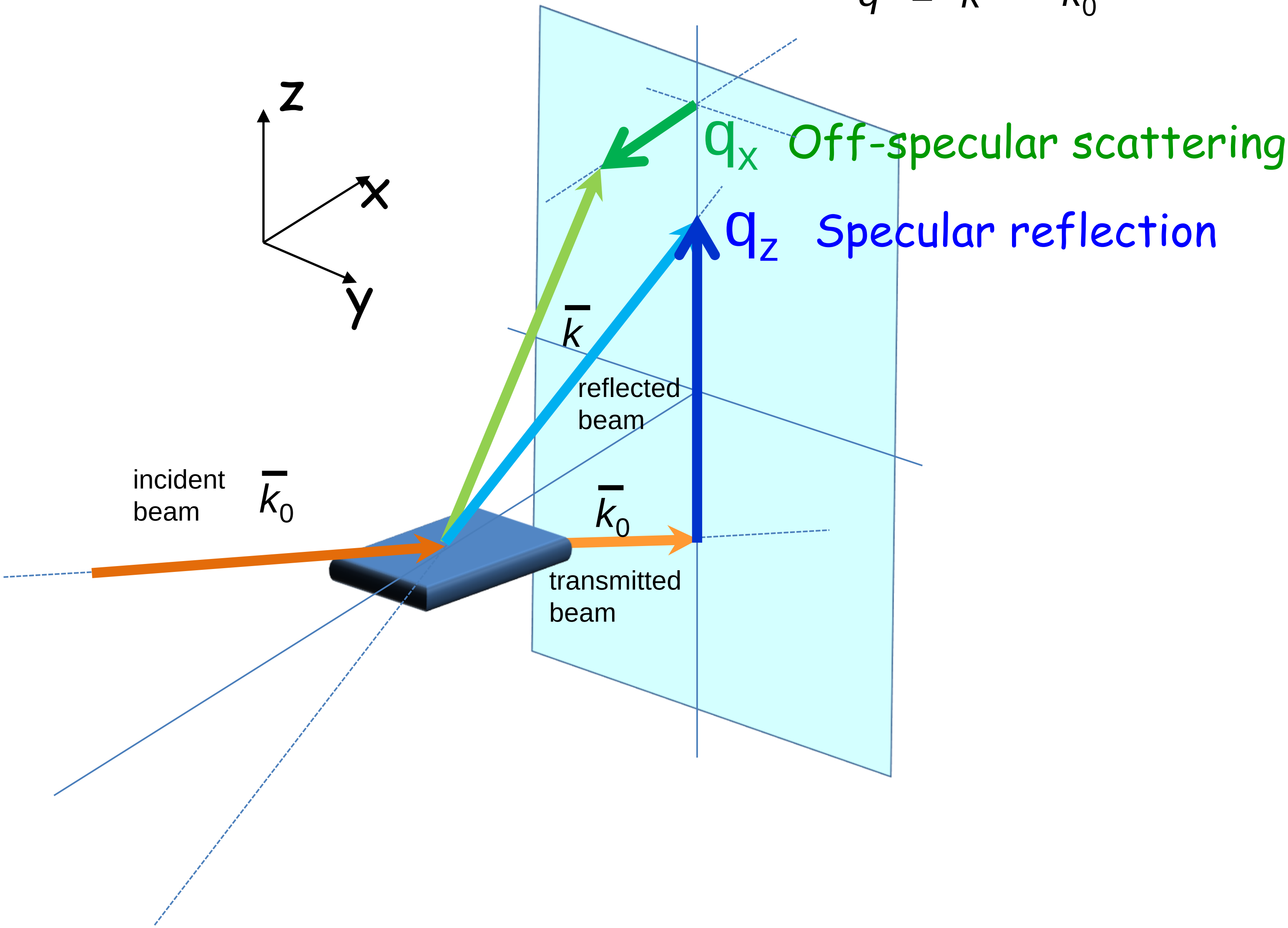
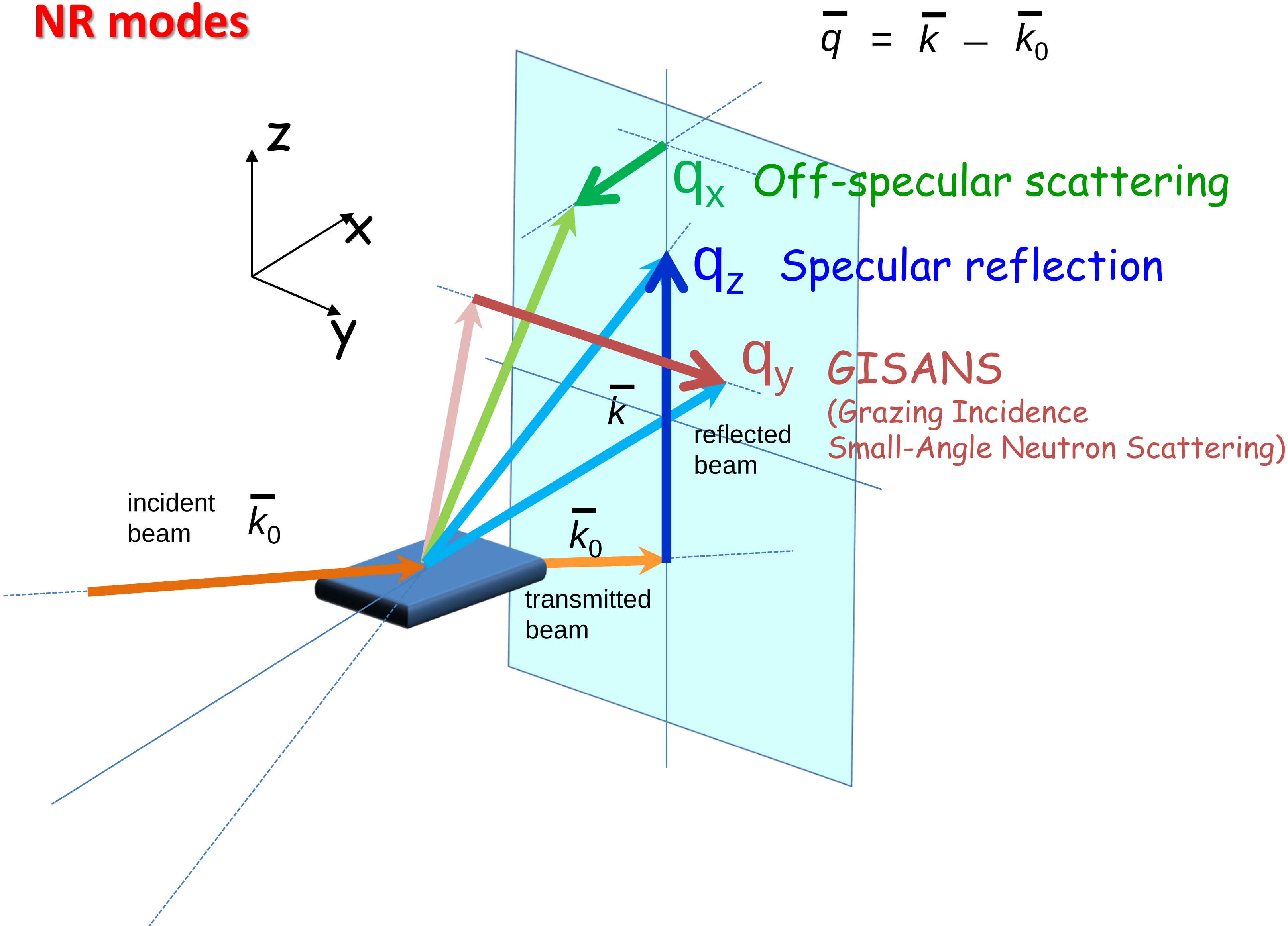
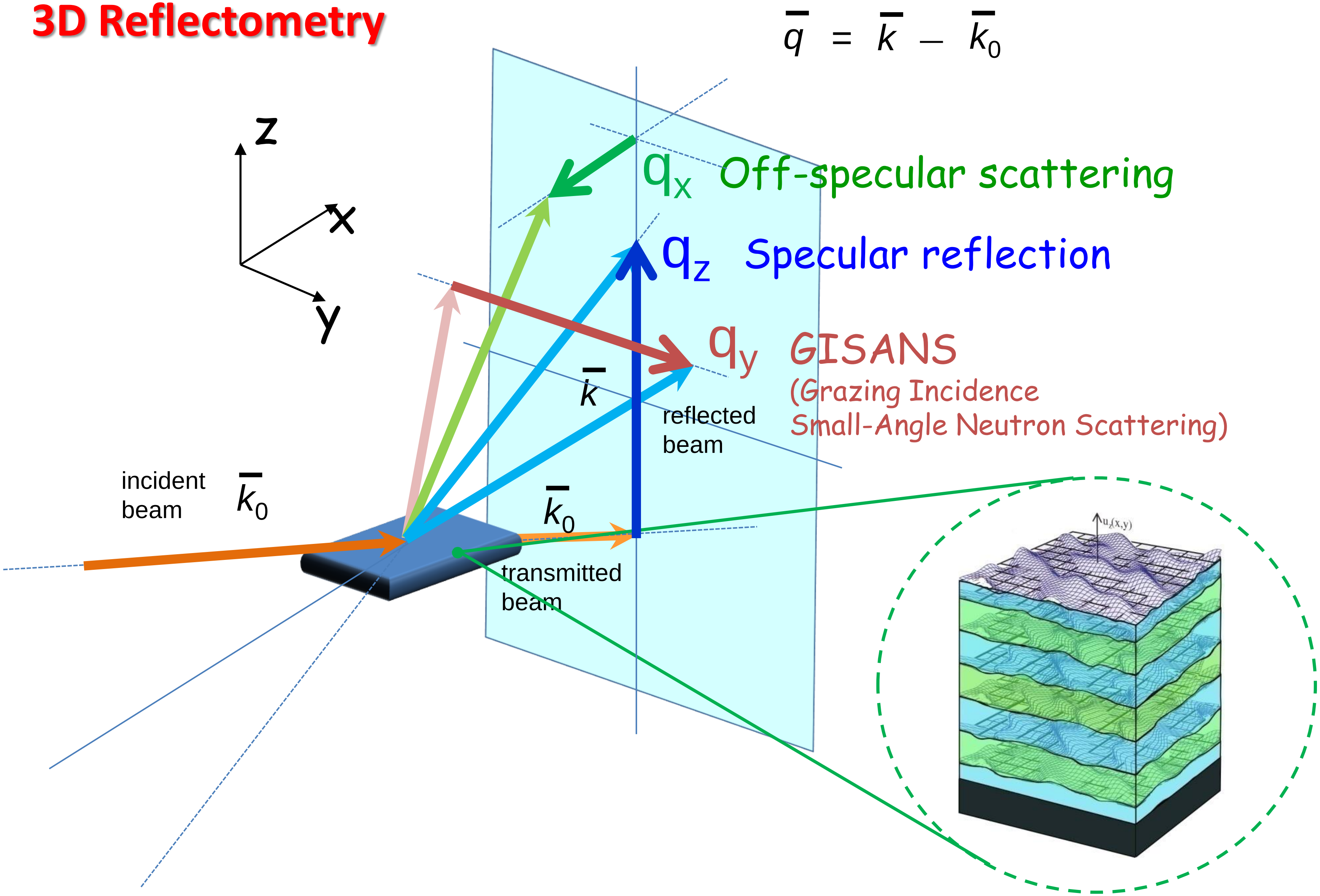


NR modes



$\bar{q} = \bar{k} - \bar{k}_0$
z
x
y
q_x Off-specular scattering
q_z Specular reflection
q_y GISANS
(Grazing Incidence
Small-Angle Neutron Scattering)
$\bar{k}$
reflected
beam
incident
beam
$\bar{k}_0$
$\bar{k}_0$
transmitted
beam

3D Reflectometry



Complete reflectometry

$q_x \, q_y \, q_z$

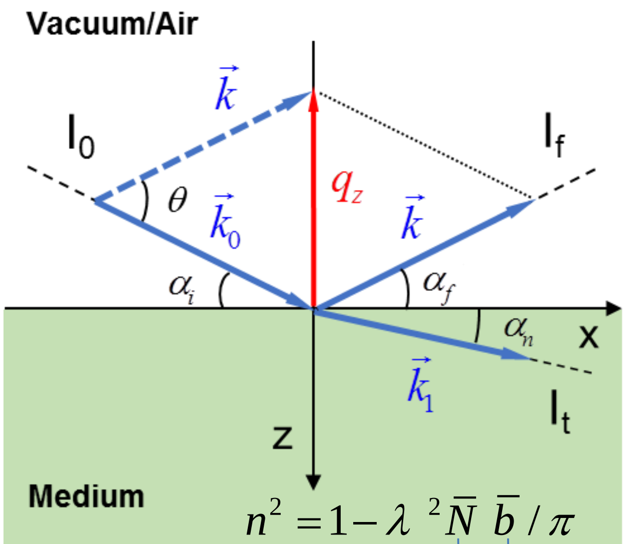
$u_z(x,y)$ – vertical surface displacement

$\xi_x, \ \xi_y$ – correlation lengths

Specular reflection

Tight collimation of incident beam over Z-direction

Optical approach

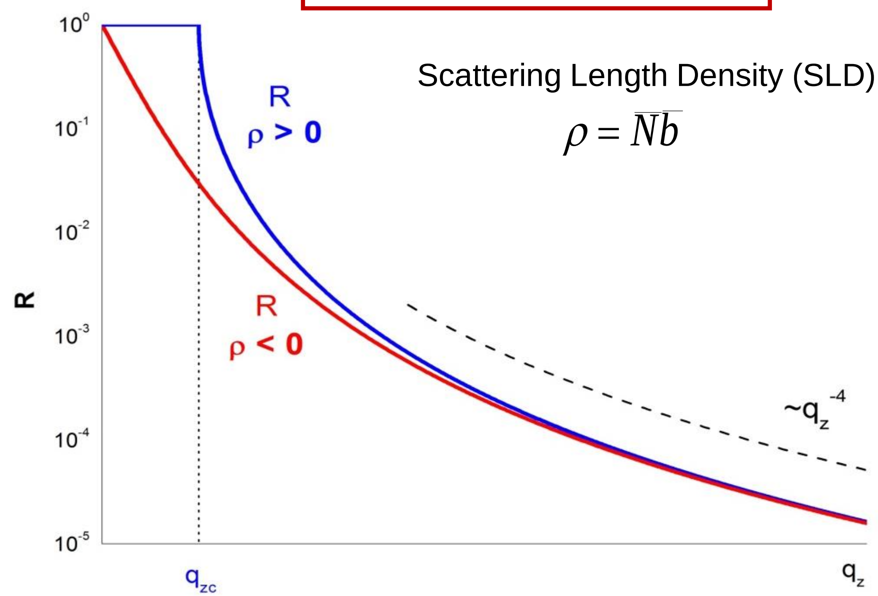


$$q_z = \frac{4\pi}{\lambda}\sin\frac{\theta}{2}$$

$$\theta = 2\alpha_i$$

$$\alpha_i \sim 1\!-\!10 \text{ mrad}$$

Reflectivity $R(q_z) = I_f/I_0$



Fresnel law

$$R(q_z) = \left|\frac{\sin\alpha_i - n\sin\alpha_n}{\sin\alpha_i + n\sin\alpha_n}\right|^2 = \left|\frac{1-\sqrt{1-q_{zc}^2/q_z^2}}{1+\sqrt{1-q_{zc}^2/q_z^2}}\right|^2$$

$$q_{zc} \approx 4(\pi\rho)^{1/2}$$

At $q_z \to \infty$ $\qquad R(q_z) \approx (1/16)(q_{zc}^4/q_z^4)$

Total external reflection

$R = 1$ at $\alpha_i < \alpha_c$ $\qquad \cos\alpha_c = n$

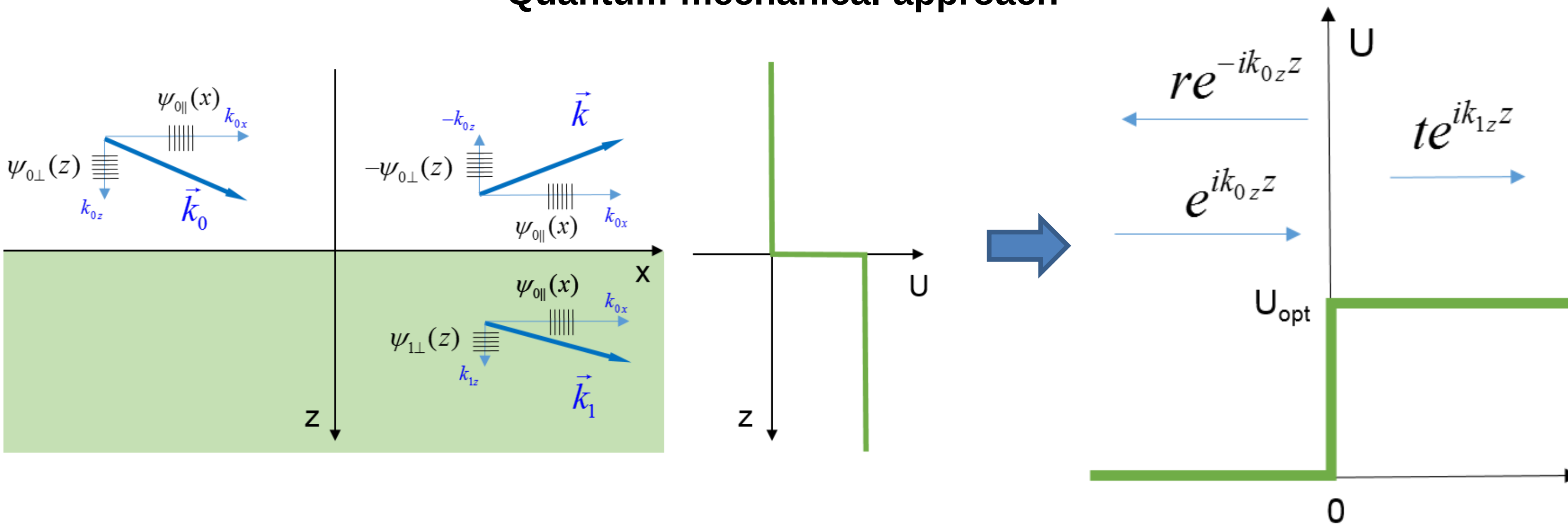
$\alpha_c^2 \approx 1 - n^2 \quad \rightarrow \quad \alpha_c = \lambda\sqrt{\dfrac{\bar{N}\,\bar{b}}{\pi}}$

for ^{58}Ni $\quad (\alpha_c/\lambda) \approx 0{,}0017$ rad/Å

Specular reflection

Tight collimation of incident beam over Z-direction

Quantum-mechanical approach



$$r = (k_{0z} - k_{1z})/(k_{0z} + k_{1z}),$$
$$t = 1 + r = 2k_{0z}/(k_{0z} + k_{1z}).$$

$$q_{zc} = 2\sqrt{2mU_{opt}/\hbar^2}$$

$$U_{opt} = 2\pi\hbar^2\rho/m$$

$$k_0\cos\alpha_i = k_1\cos\alpha_n$$

$$n = k_1/k_0 = v_1/v_0$$

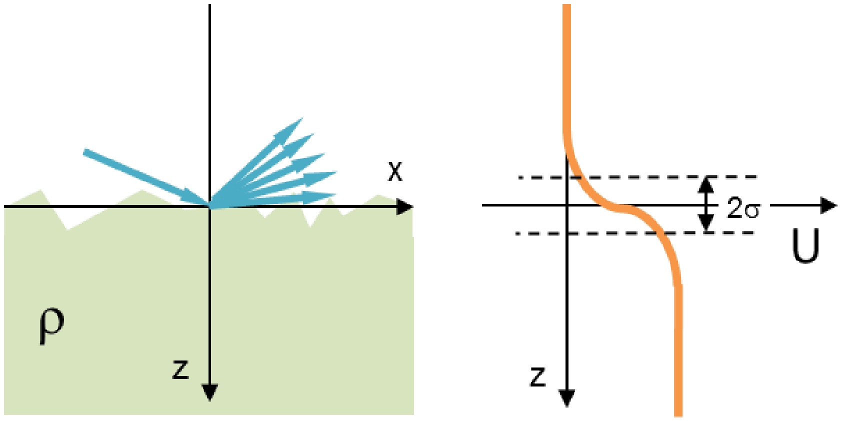
$$k_{0z} = \sqrt{2mE_\perp/\hbar^2},$$
$$k_{1z} = \sqrt{2m(E_\perp - U_{opt})/\hbar^2}$$

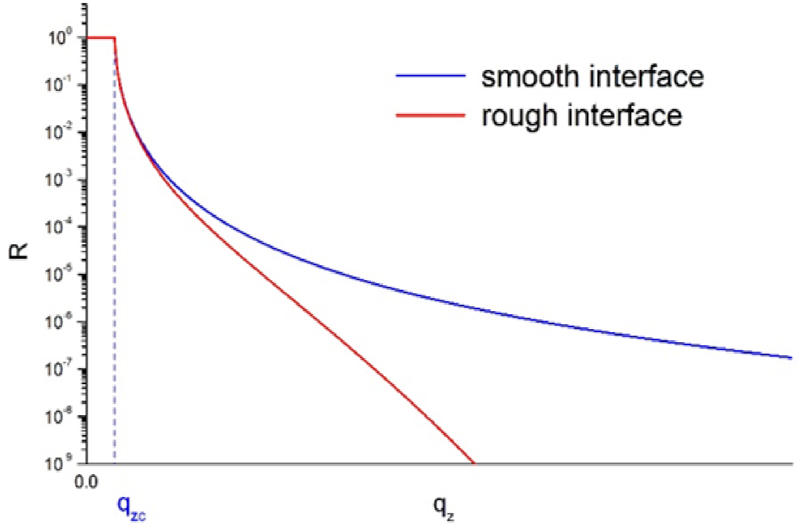
$$R = |r|^2 = \left|\frac{k_{0z} - k_{1z}}{k_{0z} + k_{1z}}\right|^2 = \left|\frac{1 - \sqrt{1 - U_{opt}/E_\perp}}{1 + \sqrt{1 - U_{opt}/E_\perp}}\right|^2 \quad E_\perp > U_{opt}, \text{ or } q_z > q_{zc}$$

Off-specular scattering

Rough interface $$G(z) = (1/\sqrt{2\pi}\sigma)\exp[-z^2/(2\sigma^2)]$$

$$R(q_z) = \exp(-4z^2 k_{0z}^2 k_{1z}^2/\sigma^2) R_F(q_z)$$





Roughness should be as minimal as possible!

Off-specular scattering

Correlated roughness $< U_j(x,y)U_j(x',y') >= \sigma^2 e^{-\frac{\tau}{\xi}2H}, \tau = [(x-x')^2 + (y-y')^2]^{\frac{1}{2}}$

- The Hurst parameter, H = 3 –D (D is the surface fractal dimension) is varied between 0 and 1.
- The lateral correlation length ξ acts as a cut-off for the lateral length scale on which an interface begins to look smooth. If $\xi \gg \tau$ the surface is smooth.

Distorted-wave Born approximation (DWBA)

non-distorted

$$[\mathbf{D}_0 - 4\pi\hat{v}(\mathbf{r})]\Psi(\mathbf{r}) = 0 \qquad \mathbf{D}_0 := \nabla^2 + K^2 \qquad v(\mathbf{r}) := \frac{m}{2\pi\hbar^2}V(\mathbf{r}) = \sum_j \langle b_j \delta[\mathbf{r} - \mathbf{r}_j(t)]\rangle$$

distorted

$$\boxed{\hat{v}(\mathbf{r}) = \bar{v}(z) + \hat{u}(\mathbf{r})} \qquad \mathbf{D} := \mathbf{D}_0 - 4\pi\bar{v}(z) \qquad \mathbf{D}(z)\Psi(\mathbf{r}) = 4\pi\hat{u}(\mathbf{r})$$

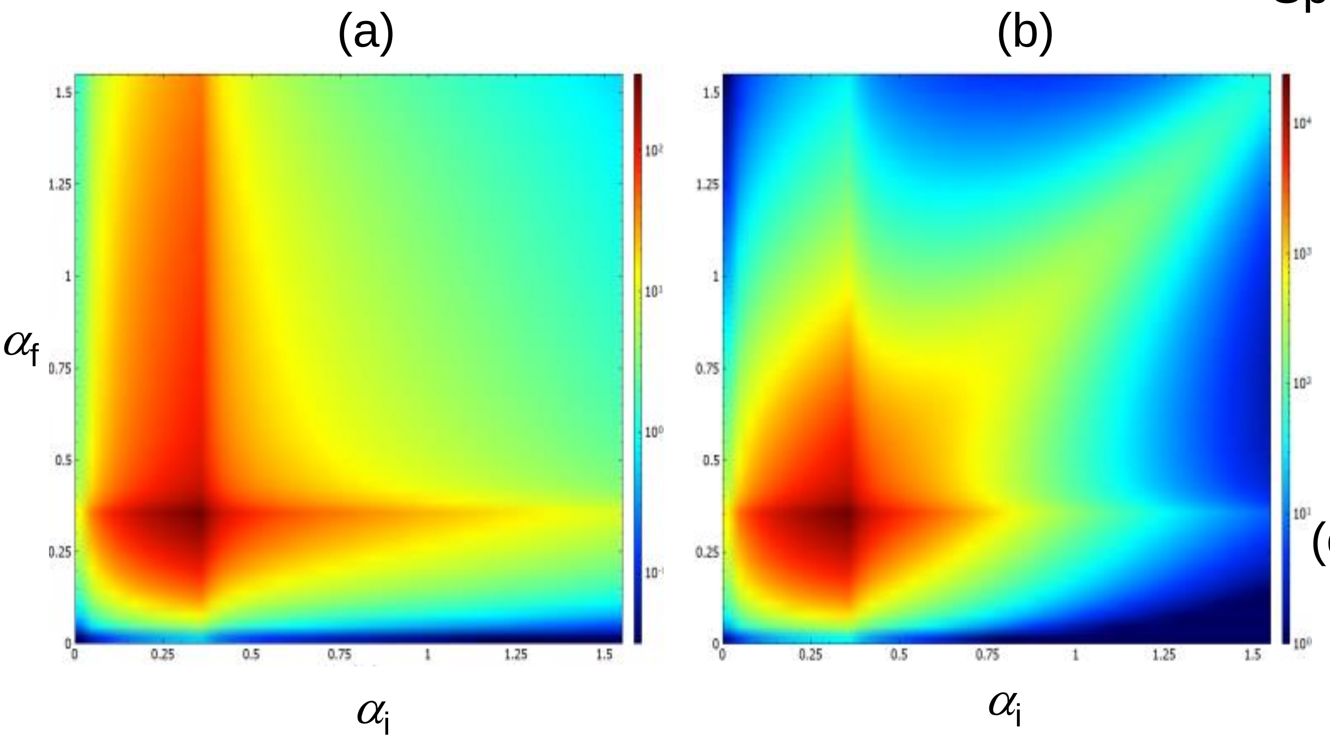
In-plane function

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left| \int \mathrm{d}^3 r \, \Psi_\mathrm{i}(\mathbf{r})\hat{u}(\mathbf{r})\Psi_\mathrm{f}^*(\mathbf{r}) \right|^2 \qquad \mathbf{D}(z)\Psi(\mathbf{r}) = 0$$

BornAgain program software

G. Pospelov, et al. J. Appl. Cryst. 53 (2020) 262–276

Off-specular scattering



Specular reflection line

$\alpha_i = \alpha_f$

Yoneda wings

(evanescent wave)

$\alpha_i = \alpha_{ic}$

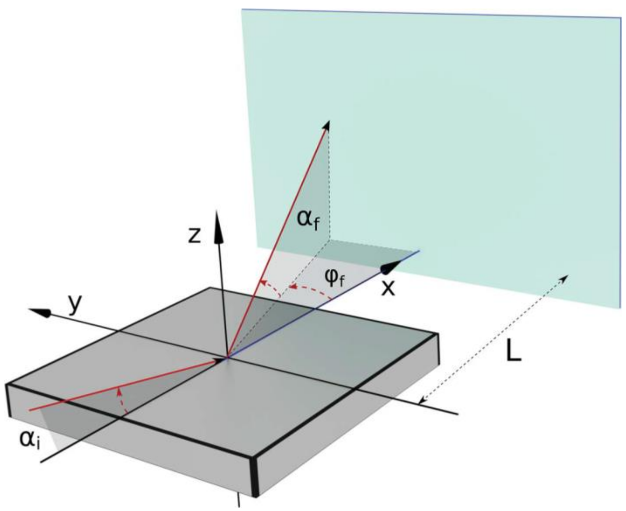
$\alpha_f = \alpha_{ic}$

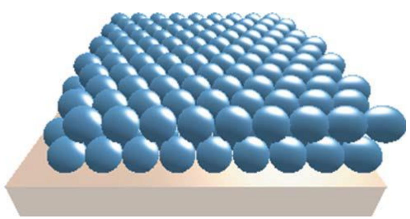
Calculated 2D patterns of NR diffuse scattering from semi-infinite medium of Cu, interface roughness σ = 2 nm,
in-plane roughness correlation length $\xi_{||}$ = 100 nm (a), $\xi_{||}$ = 1000 nm (b).

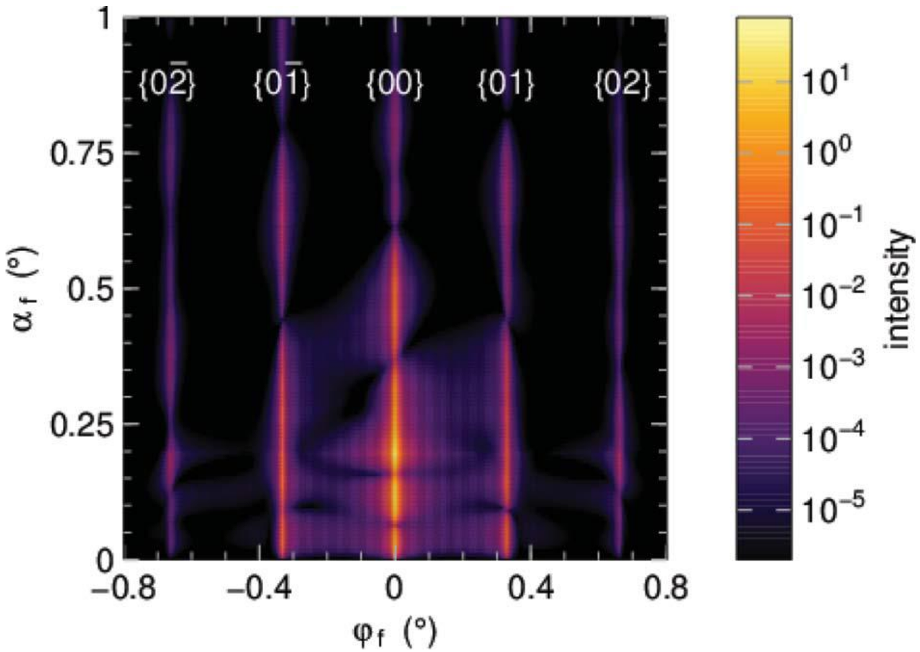
Calculations have been made in the Distorted Wave Born approximation (DWBA) using Program BornAgain; https://www.bornagainproject.org/

Grazing incidence small-angle scattering

Tight collimation of incident beam over Z- and Y- directions







R = 10 nm, hexagonal lattice (orientation ({11}, along x)).
$\xi_{||}$ = 400 nm; n = 1 - δ; $\delta = 6 \times 10^{-4}$; substrate $\delta = 6 \times 10^{-6}$;
λ = 1 A °, $\alpha_i = 0.2^{o}$.

Calculations have been made in the Distorted Wave Born approximation (DWBA) using Program BornAgain;
https://www.bornagainproject.org/
G. Pospelov, et al. J. Appl. Cryst. 53 (2020) 262–276

Steady-state and TOF modes

Steady-state (SS) mode

λ is fixed (monochromatization)

θ-scan (grazing angle)

$$q = \frac{4\pi}{\lambda}\sin\theta$$

Time-of-flight (TOF) mode

θ is fixed

λ-scan

$$q = \frac{4\pi}{\lambda}\sin\theta$$

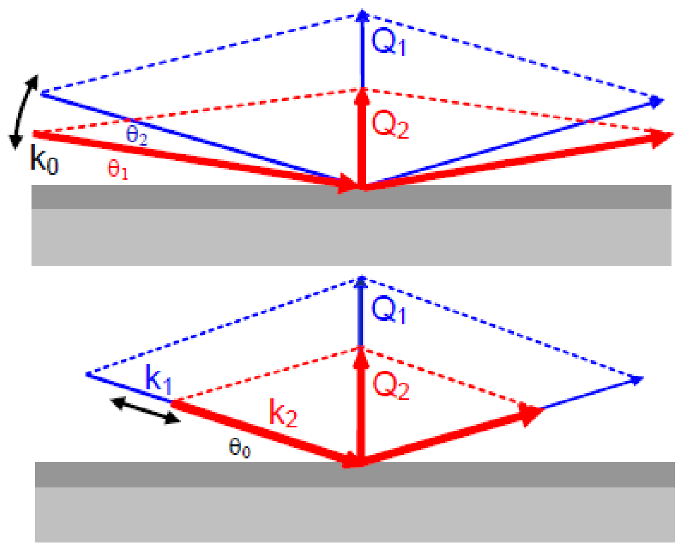
$$\lambda = \frac{h\,TOF}{mL}$$

time-of-flight

flight path

monochromatic beam

TOF



Summary to
'Principles of Neutron Reflectometry'

- ***Neutron reflectometry experiment is aimed at determining the scattering length density distribution at planar interfaces.***

- ***Two equivalent approaches to treat specular neutron reflectivity from planar interfaces exist. Optical approach is an extension of the laws of the geometrical optics of light to the case of neutrons. In quantum mechanical approach, the reflectivity is derived by considering neutron wave functions meeting barriers of the optical potential.***

- ***Off-specular scattering occurs for rough interfaces.***
  It reduces the specular reflectivity. The distribution of off-specular scattering is sensitive to in-plane (lateral) correlations.

- ***Lateral ordering of nanoscaled objects at interfaces produces 2D diffraction patterns (widened Bragg rods) in GISANS plane***

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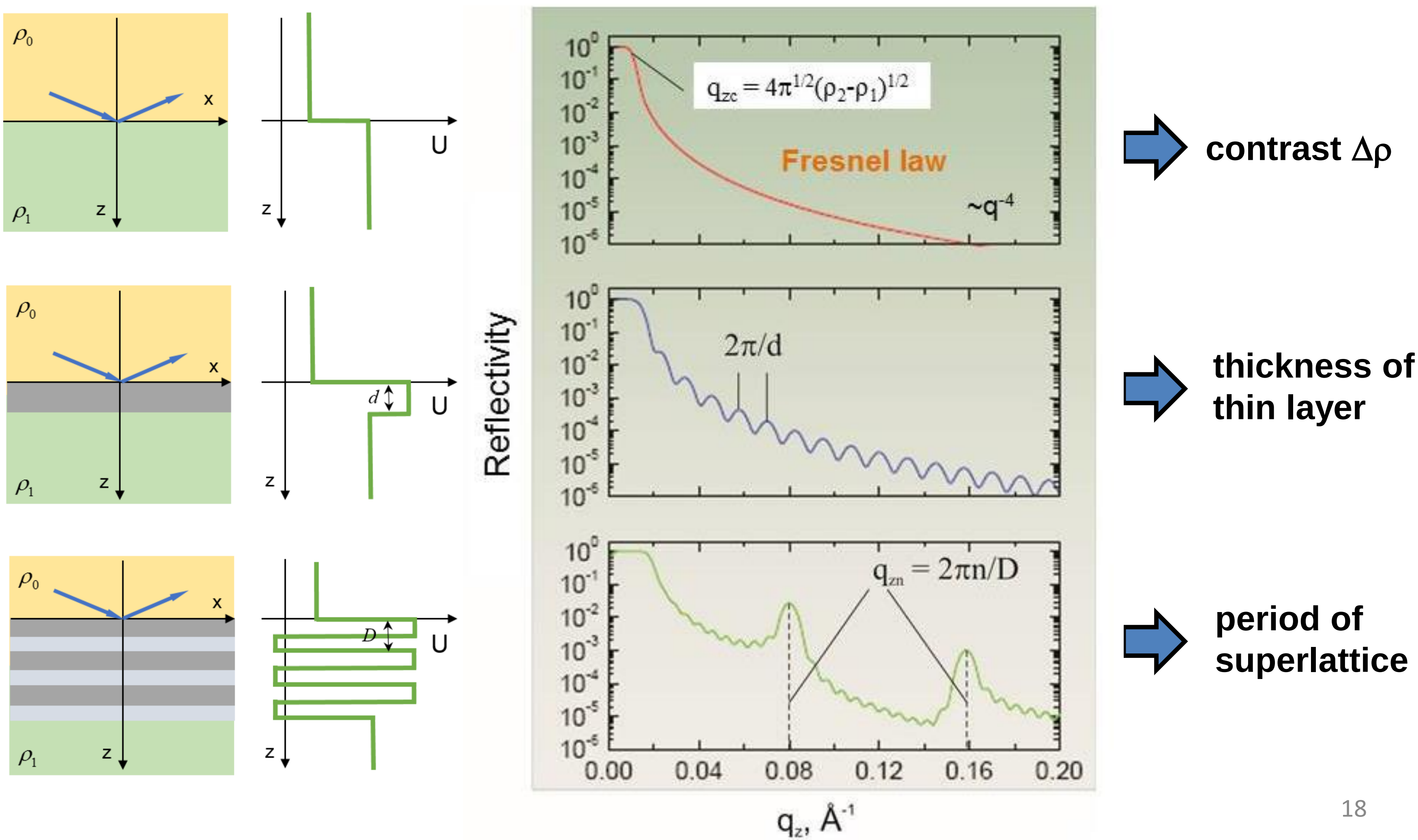
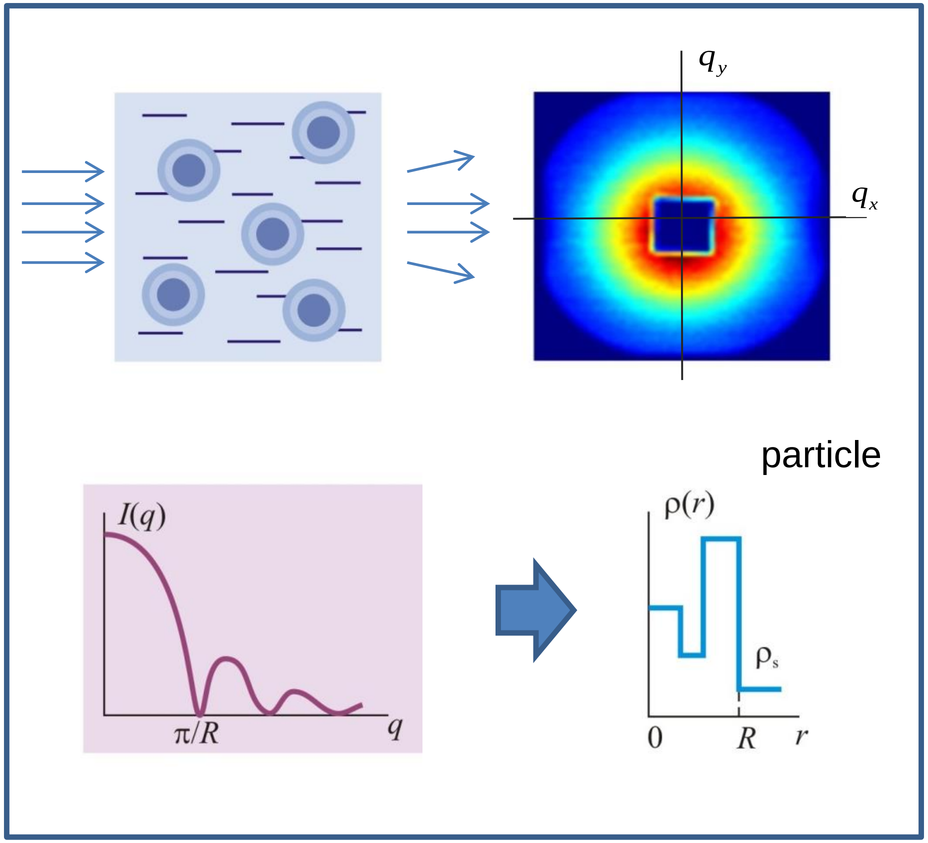
Reflectivity of arbitrary interface

Born approximation $$R(q_z) \approx \frac{16\pi^2}{q_z^4}\left|\int_{-\infty}^{+\infty}\left(\frac{d}{dz}\rho(z)\right)e^{-iq_z z}dz\right|^2 \qquad q >> q_{zc}$$

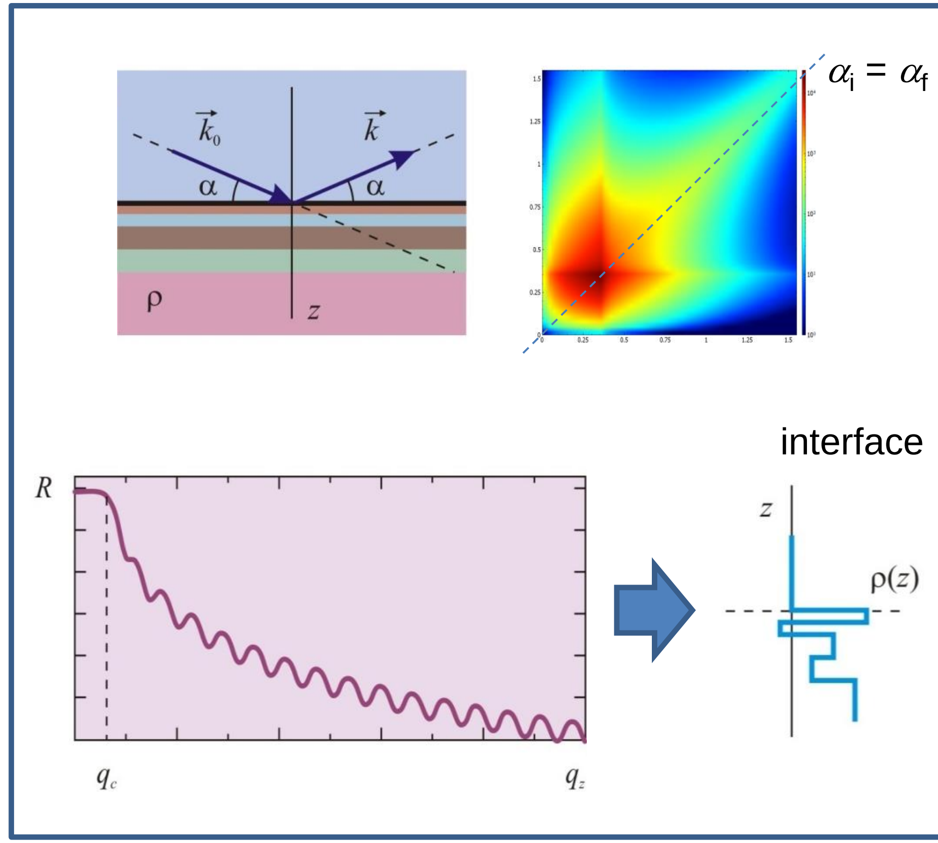


Small-angle Scattering and Reflectometry

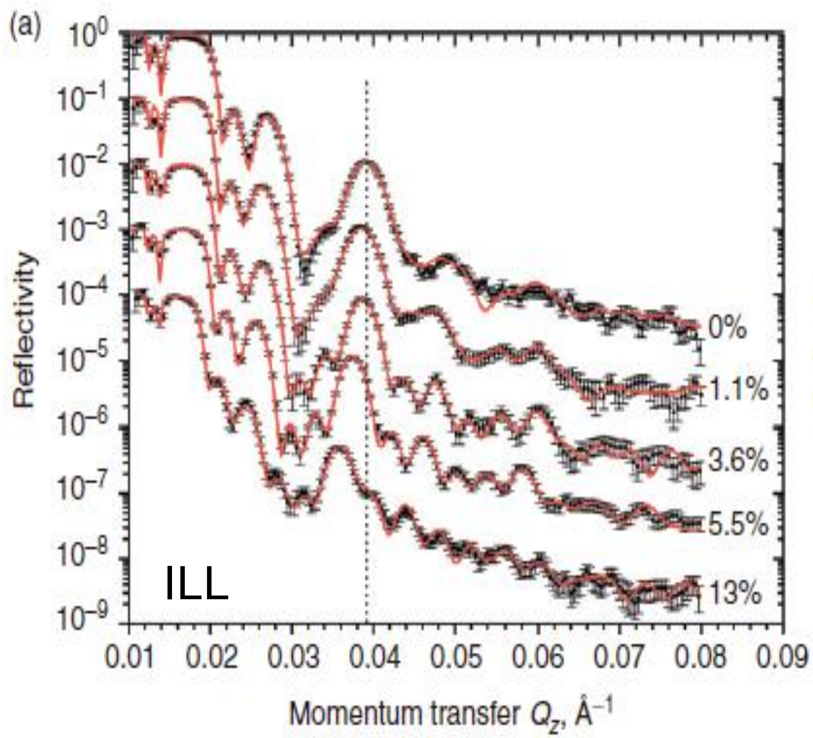
Small-angle scattering (SAXS, SANS)
- **bulk structure**

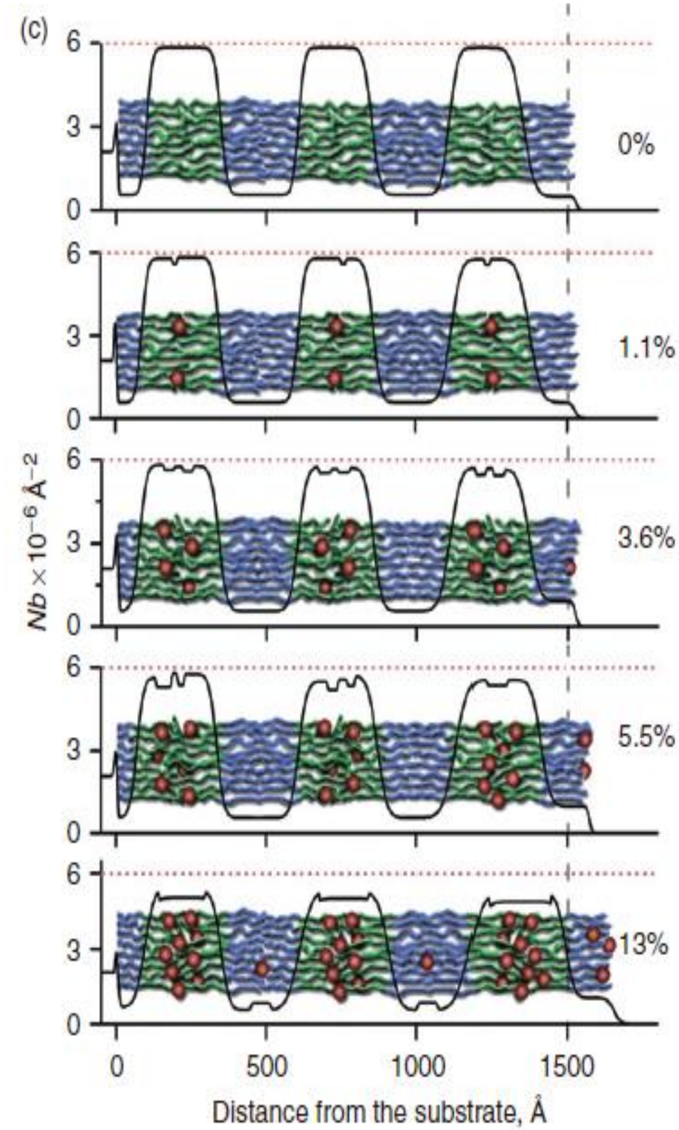


Reflectometry (XRR, NR)
- **structure at interface**



Reflectivity from composite films

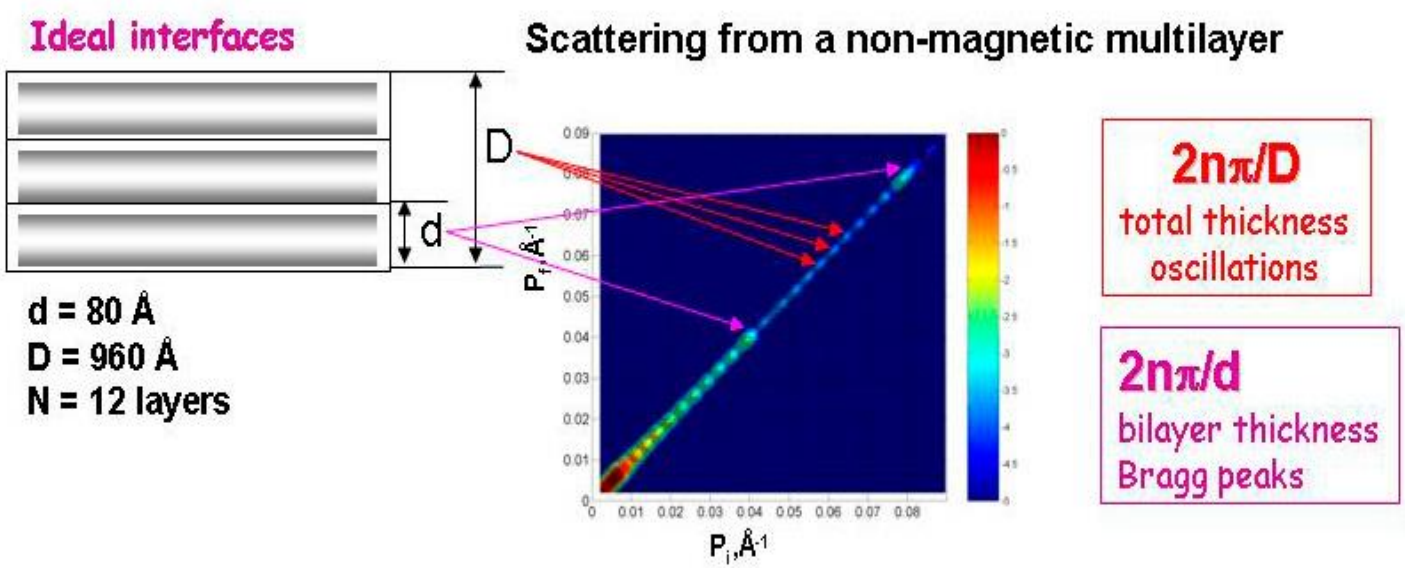


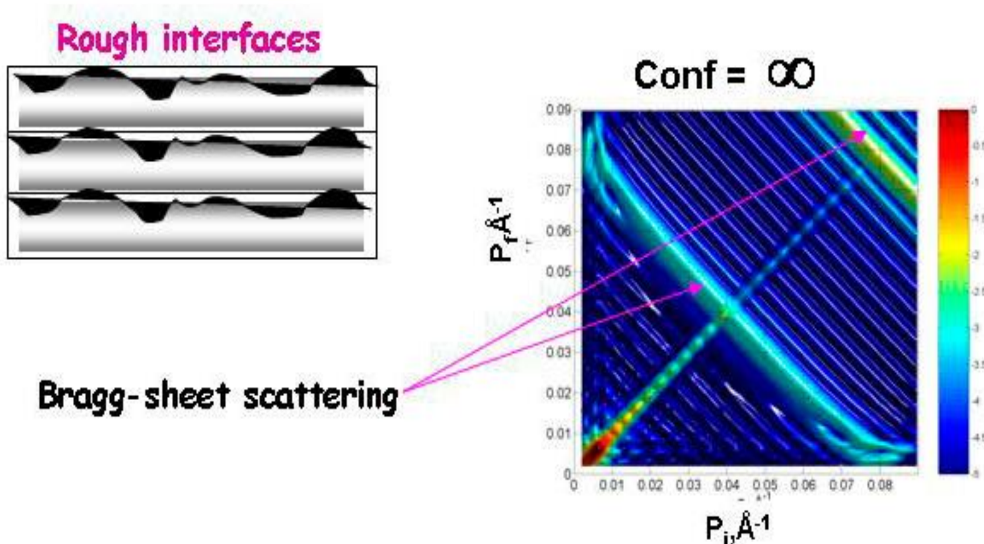


Block copolymer (PS-d-b-PBMA) multilayers mixed with nanoparticles (Fe_3O_4, D ~ 5 nm) on Si substrate.

Lauter, H.J.C., Lauter, V., Toperverg, B.P. *Polymer Science: A Comprehensive Reference* (2012) 2, 411

Off-specular scattering from multilayers





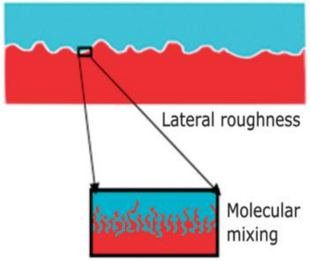
σ = 0.5 nm,
ξ_{II} = 30 nm
$\xi_{\perp} = \infty$

V. Lauter-Pasyuk, J. Phys. IV France 1 (2007)

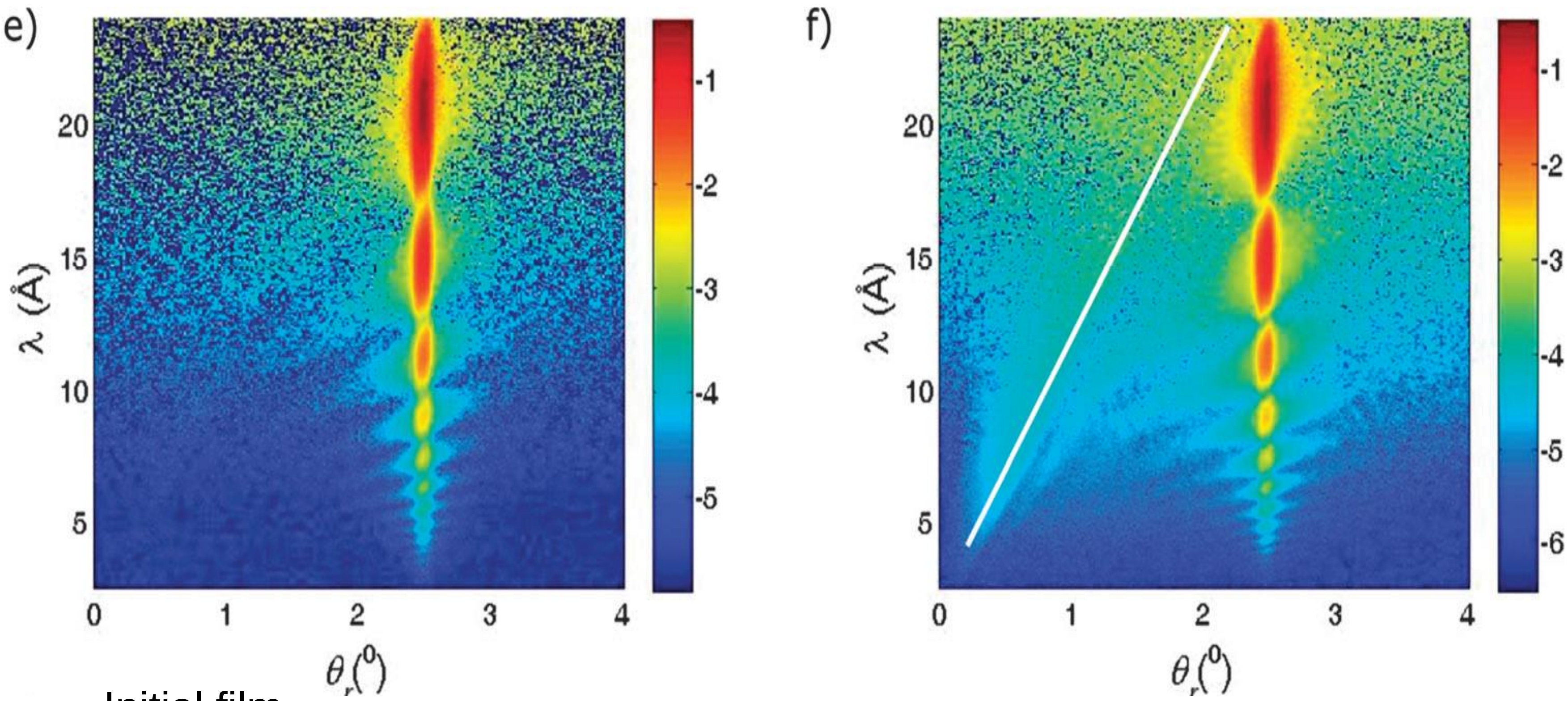
Off-specular scattering in TOF mode

Molecular mixing at a conjugated polymer interface

F8 [100 nm]/ d-PMMA [48 nm] / Si



D17, ILL



Initial film

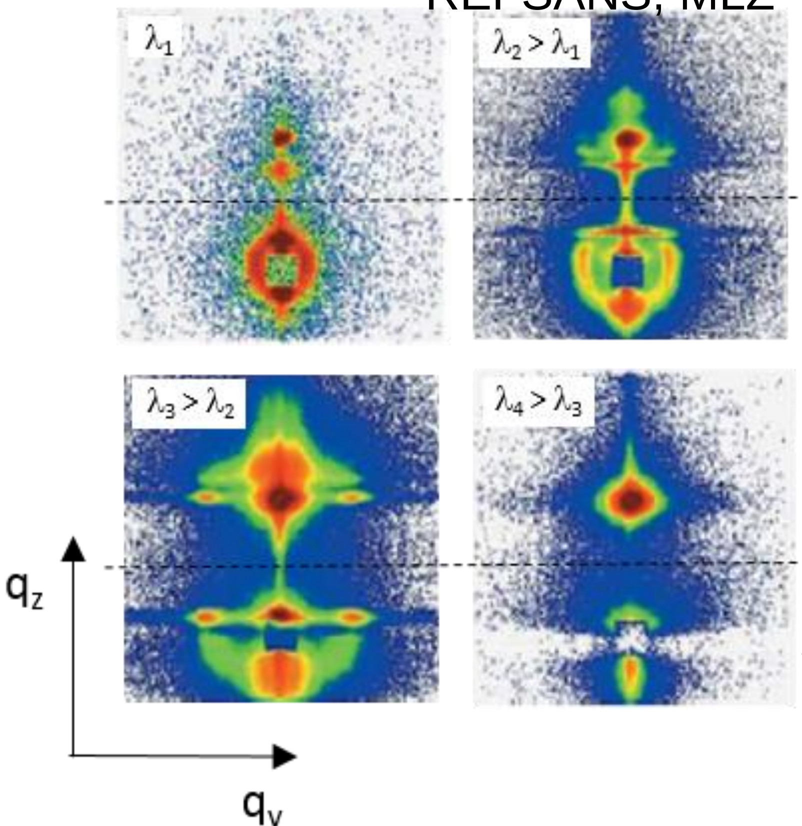
D.James, et al., *Soft Matter* **11**(48), 9393 (2015)

After annealing (Yoneda wing is marked by white line)

GISANS

Polymer P(S-b-MMAd) film (thickness 800 nm) on Si.

REFSANS, MLZ



Reflected beam

Horizon

Refracted beam

Penetration depth depends on neutron wavelength

$$z_{1/e} = \frac{\lambda}{2^{1/2}\pi(l_i + l_f)},$$

$$l_{i,f} = \left((\alpha_c^2 - \alpha_{i,f}^2) + \sqrt{(\alpha_{i,f}^2 - \alpha_c^2)^2 + (\lambda\mu/2\pi)^2}\right)^{1/2}$$

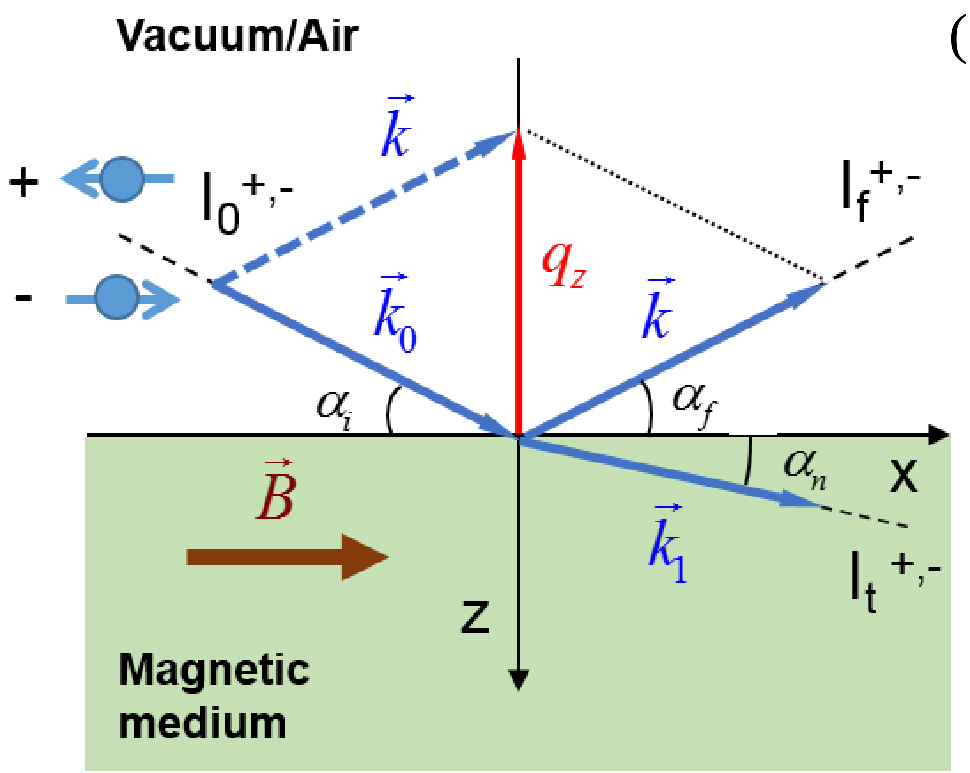
$\mu = \Sigma_{tot}$ - volume adsorption coefficient

Both reflected and refracted beams are measured!

P.Mueller-Buschbaum, et al., Eur. Phys. J. 167, 107–112 (2009)

Polarized neutrons

Specular reflection. Polarized beam.
Tight z-collimation.



Refraction index

$$(n^{\pm})^2 = 1 - [\lambda^2 Nb/\pi \pm (m/2\pi\hbar^2)\mu B]$$

Nuclear SLD $\rho_n = Nb$

Magnetic SLD $\rho_m = (m\mu B/2\pi\hbar^2)$

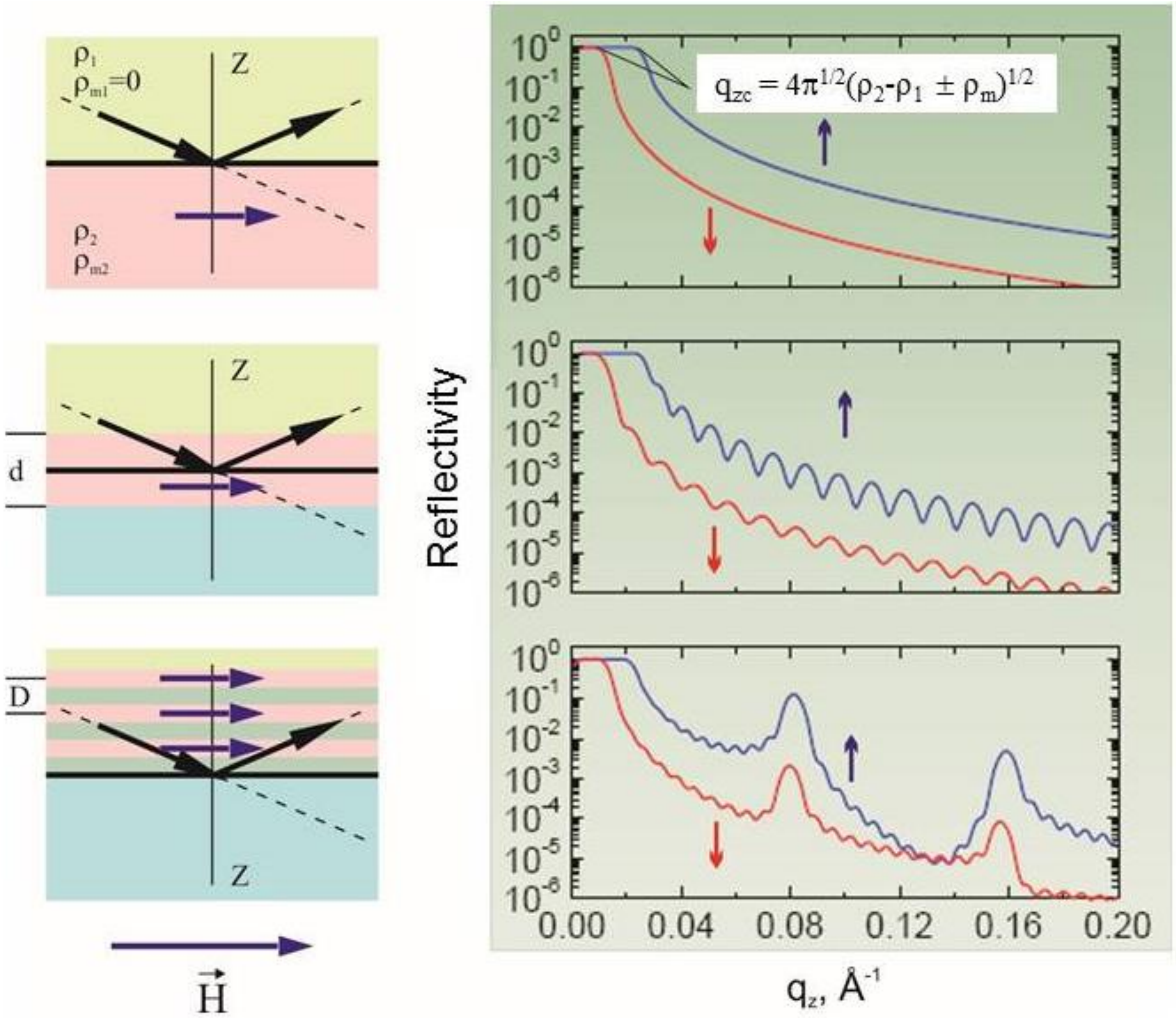
$$q_{zc}^{\pm} = (q_{zcn}^2 \pm q_{zcm}^2)^{1/2}$$

$$q_{zcn,\ zcm} = 4(\pi\rho_{n,m})^{1/2}$$

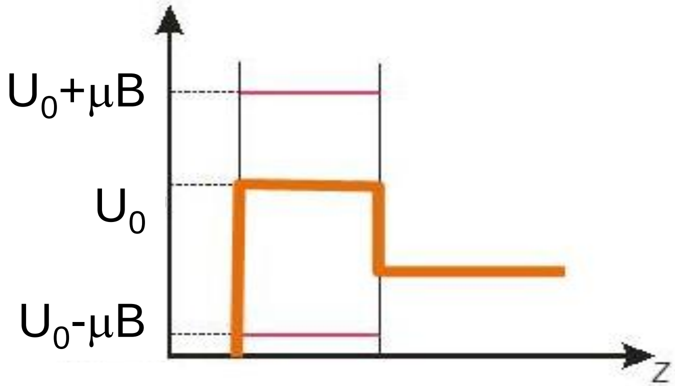
$$R^{\pm}(q_z) = \left| \frac{1 - \sqrt{1 - (q_{zc}^{\pm})^2 / q_z^2}}{1 + \sqrt{1 - (q_{zc}^{\pm})^2 / q_z^2}} \right|^2$$

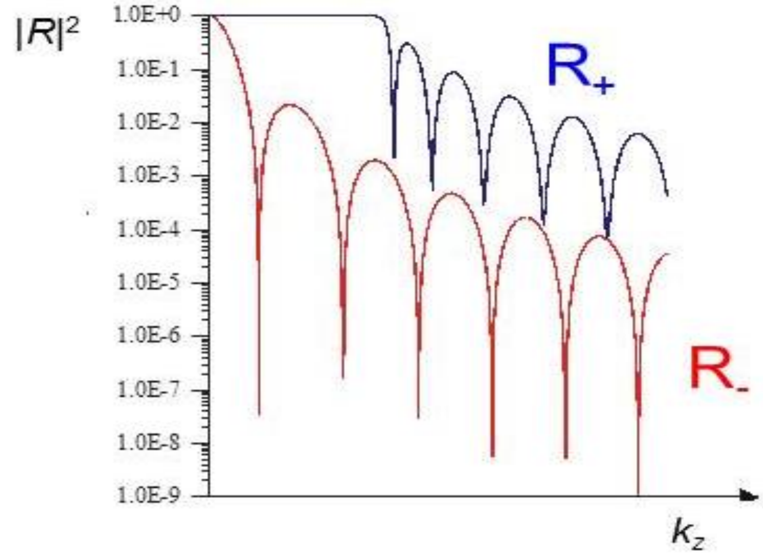
Reflectivity of arbitrary interface. Polarized neutrons

Born approximation $$R^{\pm}(q_z) = \frac{16\pi^2}{q_z^4} \left| \int_{-\infty}^{+\infty} \left(\frac{d}{dz}[\rho_n(z) \pm \rho_m(z)]\right) e^{-iq_z z} dz \right|^2 \qquad q >> q_{zc}$$

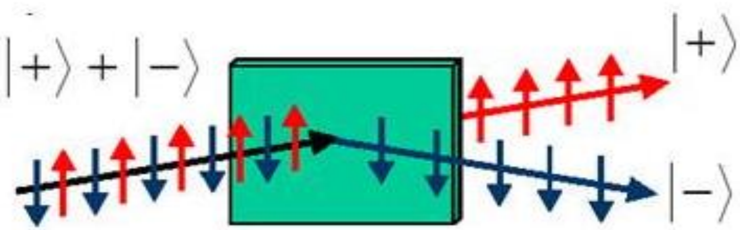


Polarization of neutron beams

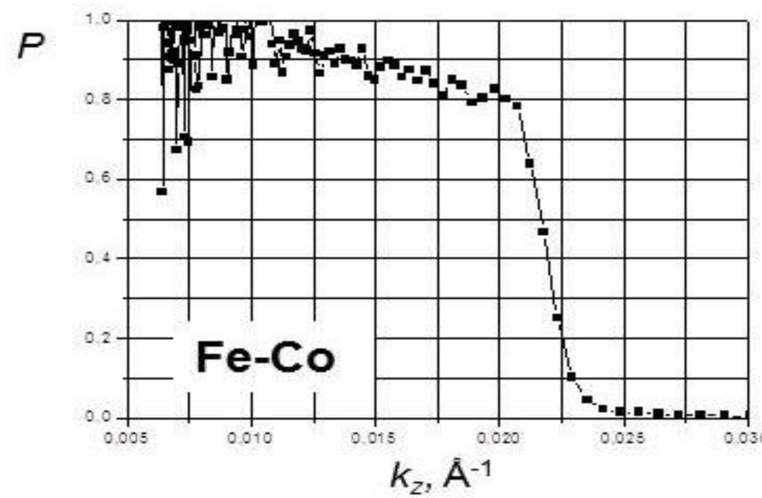




Polarization



$$P = \frac{N_+ - N_-}{N_+ + N_-} \qquad P = \frac{R_+(k) - R_-(k)}{R_+(k) + R_-(k)}$$



Off-specular scattering of polarized neutrons

Mixed states

$$\psi_{\perp i}(z) = \exp(ik_{0z}z)\begin{pmatrix}\psi_+^{(i)} \\ \psi_-^{(i)}\end{pmatrix};$$ - Incident beam

$$\psi_{\perp f}(z) = \exp(-ik_{0z}z)\begin{pmatrix}\psi_+^{(f)} \\ \psi_-^{(f)}\end{pmatrix}.$$ - Reflected beam

Coefficient of reflection ⇨

$$\hat{r} = \begin{pmatrix} r_{++} & r_{+-} \\ r_{-+} & r_{--}\end{pmatrix} \qquad \begin{pmatrix}\psi_+^{(f)} \\ \psi_-^{(f)}\end{pmatrix} = \widehat{r}\begin{pmatrix}\psi_+^{(i)} \\ \psi_-^{(i)}\end{pmatrix}$$

Solutions of the system of Schrödinger equations!

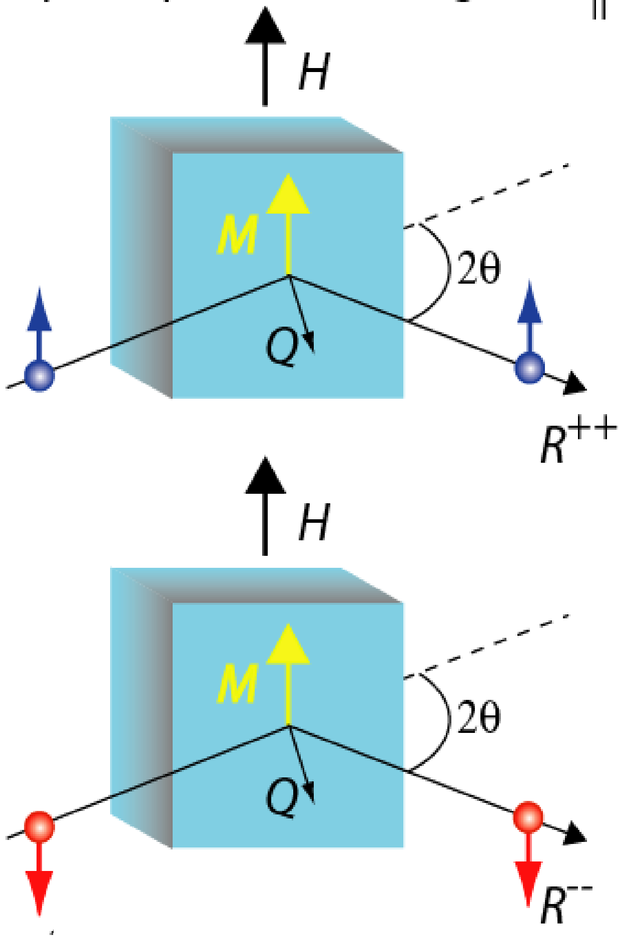
Reflectivity matrix

$$R_{++} = R_{+} = |r_{++}|^2, \quad R_{--} = R_{-} = |r_{--}|^2 \quad \}$$ reflection from in-plane *M*-component

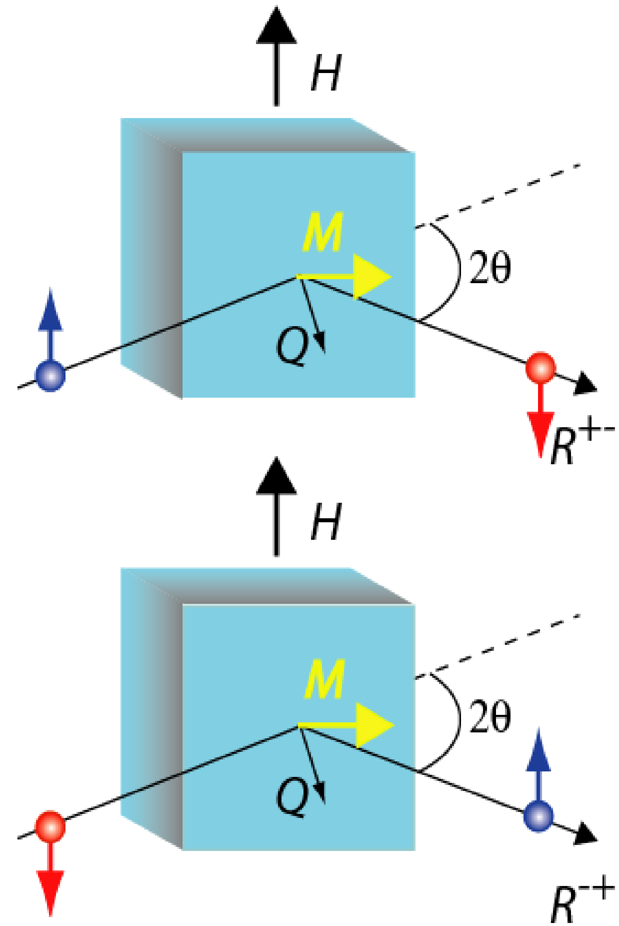
$$R_{+-} = |r_{+-}|^2, \quad R_{-+} = |r_{-+}|^2 \quad \}$$ reflection from out-of-plane *M*-component (e.g. magnetic roughness)

Full polarization analysis

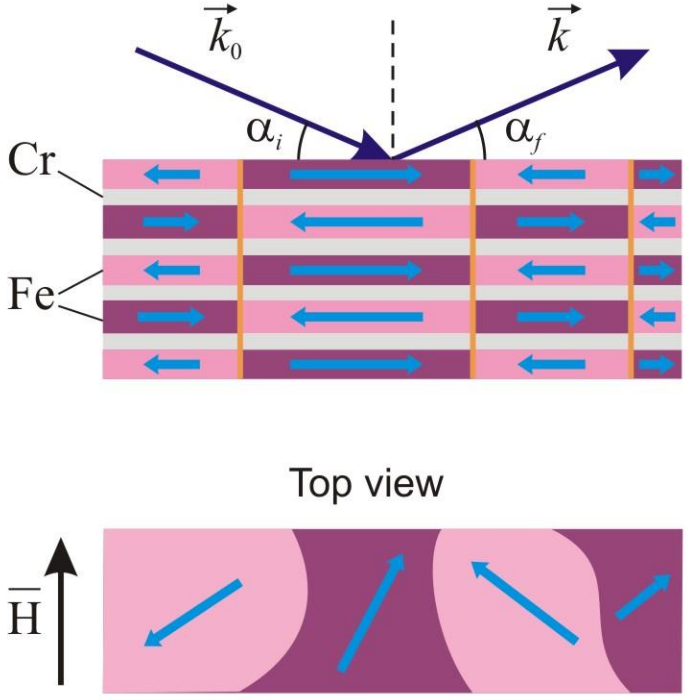
non-spin-flip reflectivities give $\overline{M_{||}(Q)}$

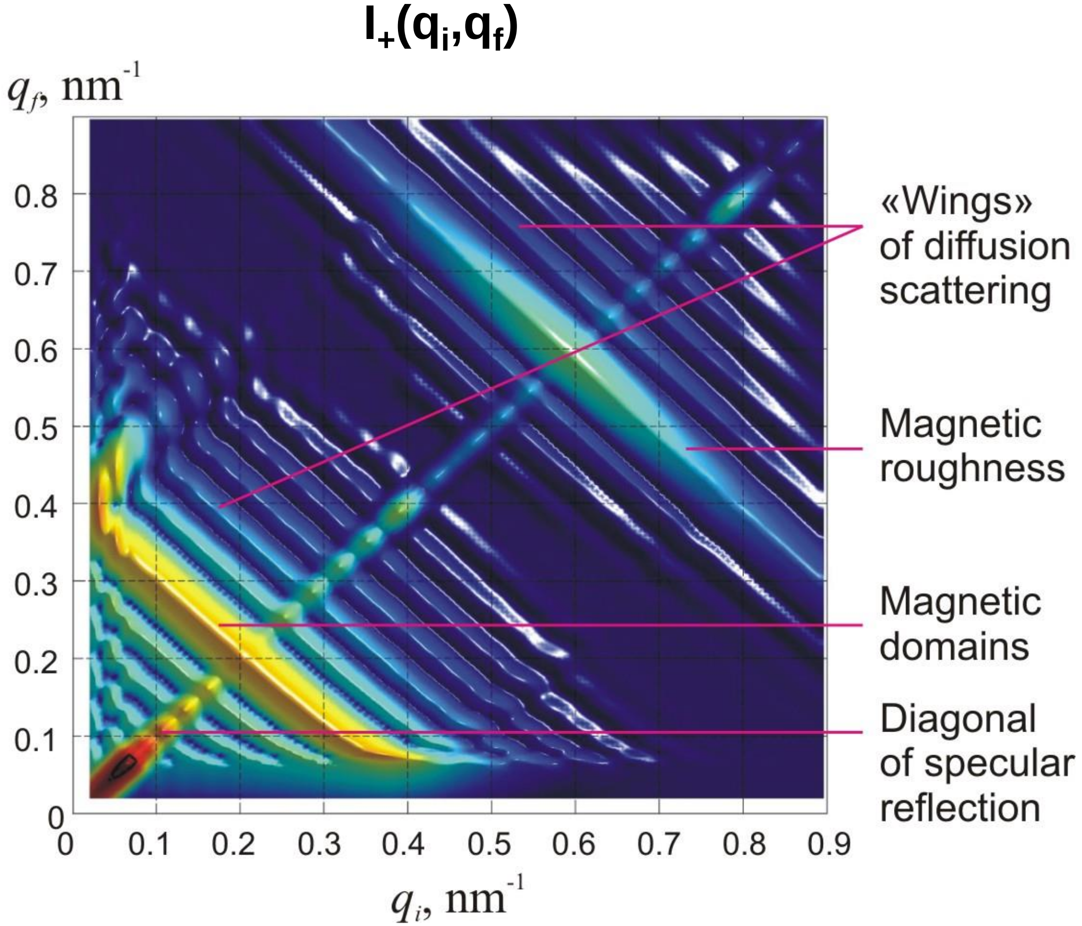


spin-flip reflectivities give $\overline{M_{\perp}^{2}(Q)}$



Magnetic multilayers





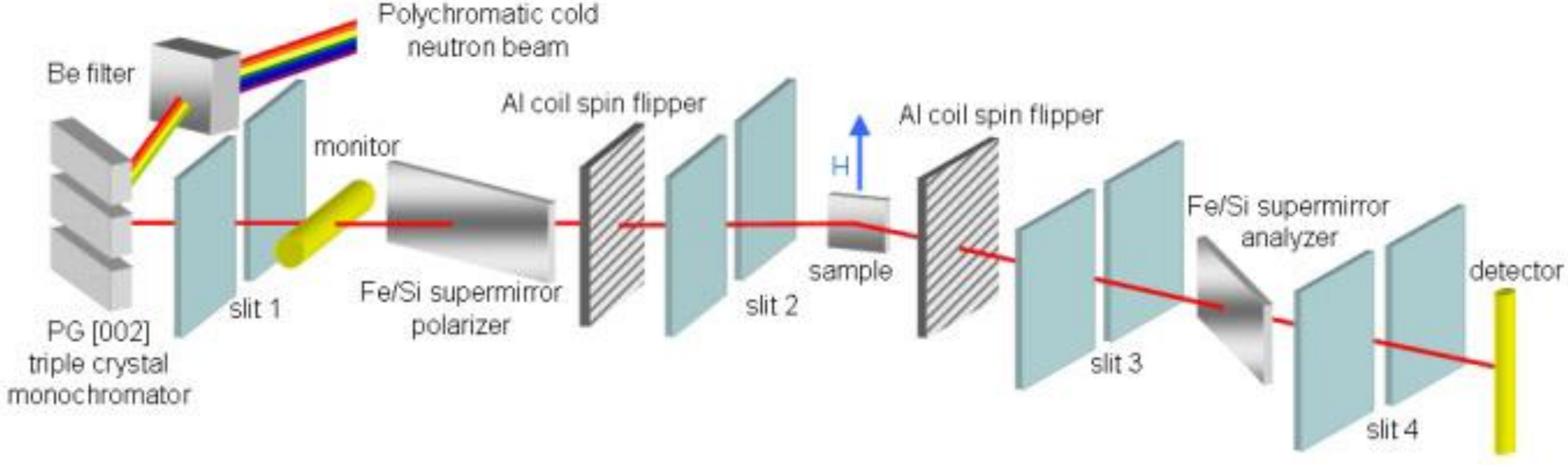
$q_i = 2k_0\sin\alpha_i$ $q_f = 2k_0\sin\alpha_f$

V. Lauter-Pasyuk, J. Phys. IV France 1 (2007)

Software: FitSuite L. Deák et al, PRB 76 224420 (2007)

Neutron reflectometer

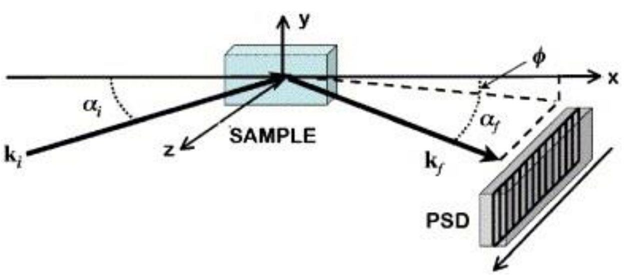
PBR polarized beam reflectometer (NIST)



Main units

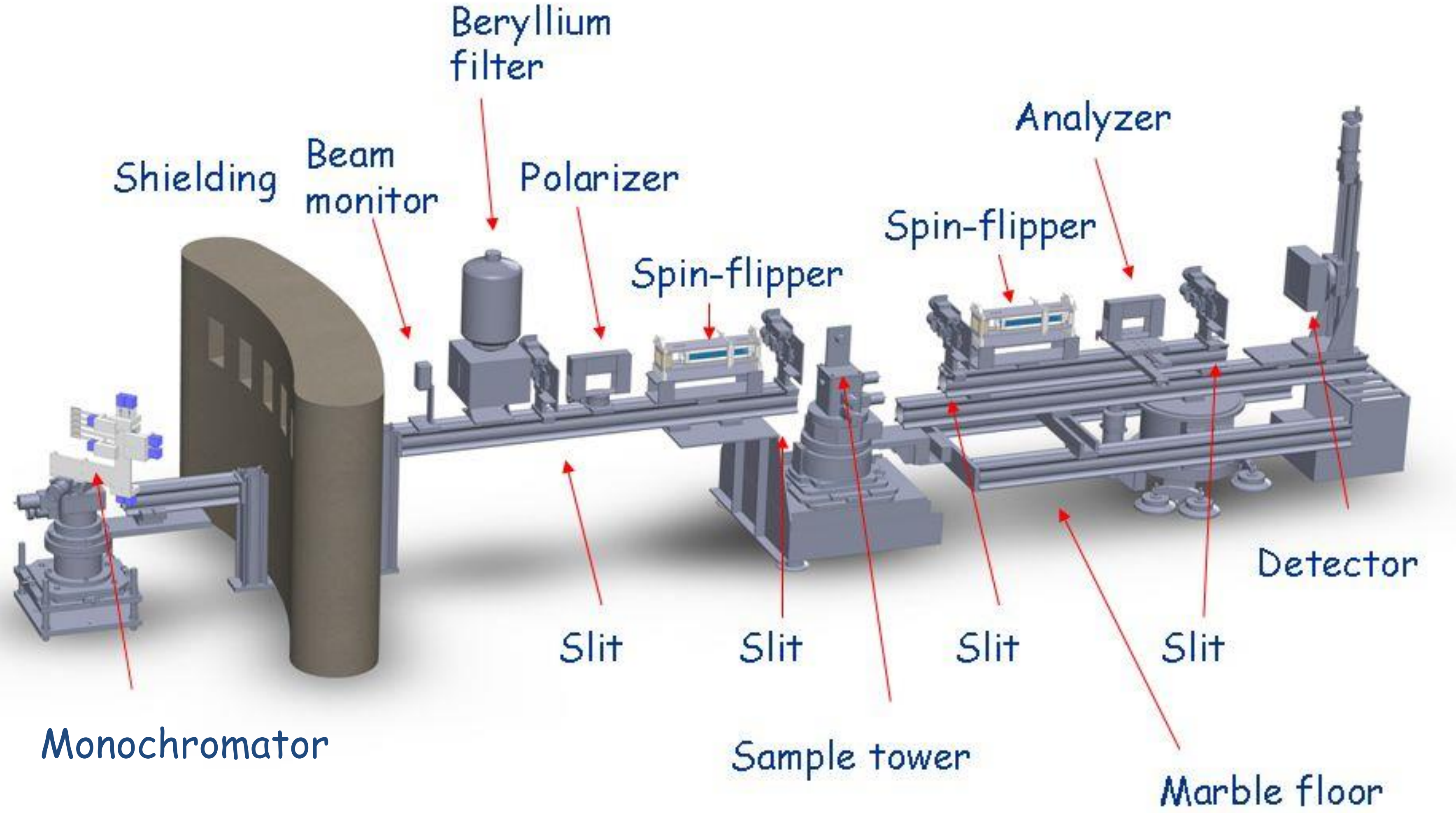
- **Polarizer** (spin polarization of neutron beam)
- **Spin-flippers** (change of beam polarization at given polarization after polarizer or sample)
- **Analyzer** (analysis of polarization after sample)
- **Detector** (detection of scattered and transmitted beams)

Vertical sample geometry



Neutron reflectometer

GINA reflectometer (BNC, Budapest)



http://mffo.rmki.kfki.hu/gina

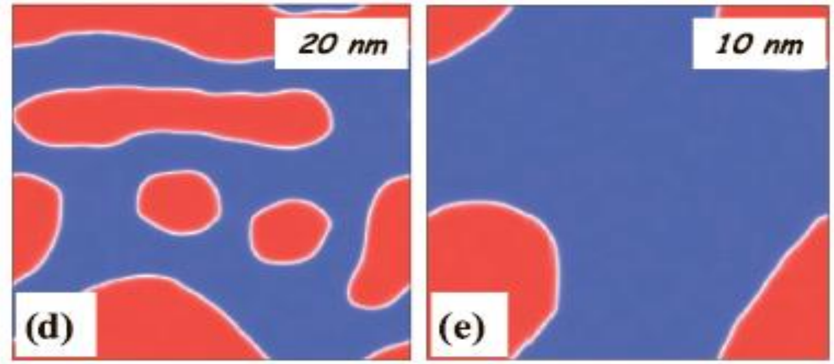
Domain structure in thin films with perpendicular anisotropy

CoCrPt films

Micromagnetic simulations

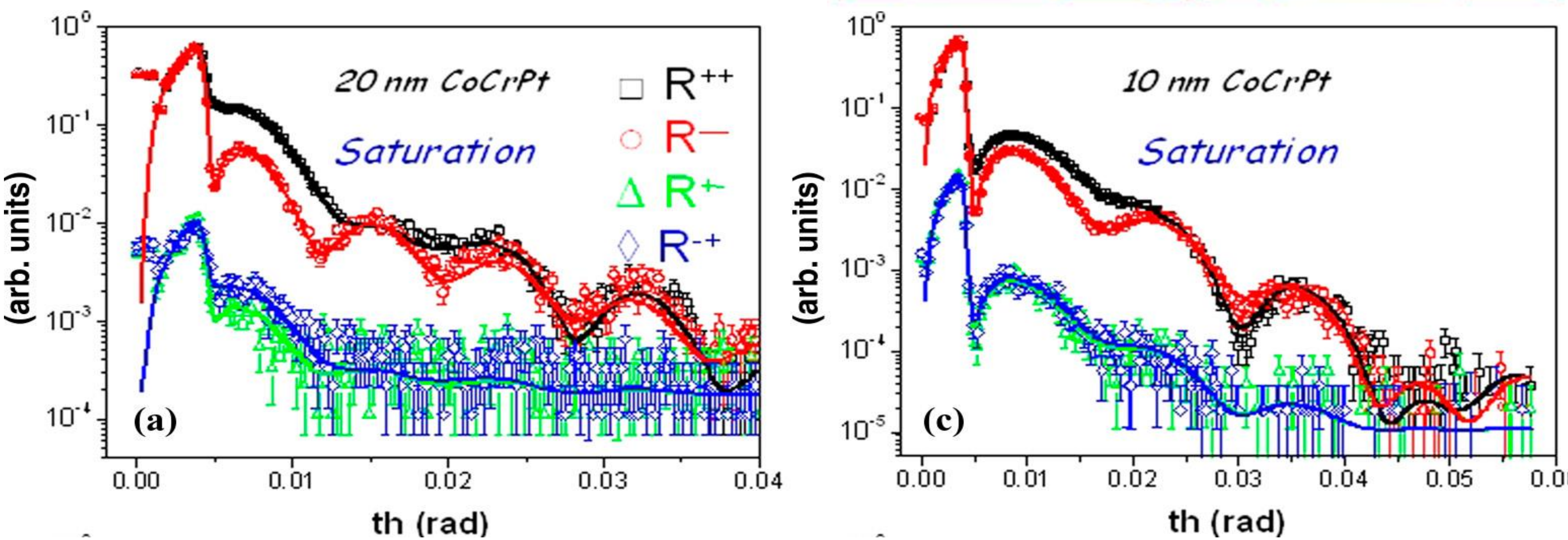
$M_z > 0$ (blue)

$M_z < 0$ (red)



SuperADAM, ILL

NR

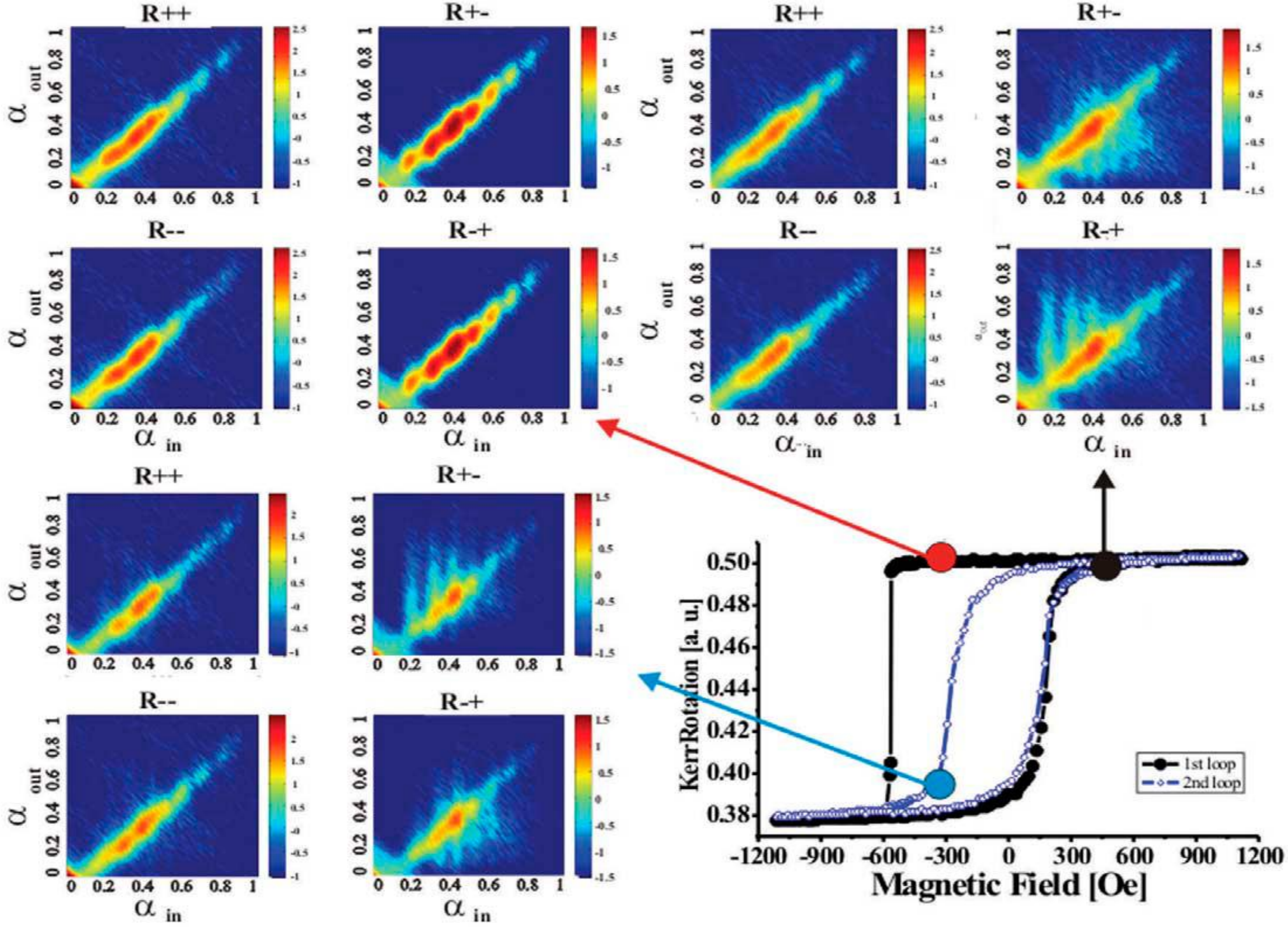


D. Navas, PRB **90**, 054425 (2014)

Magnetic bilayer with exchange bias

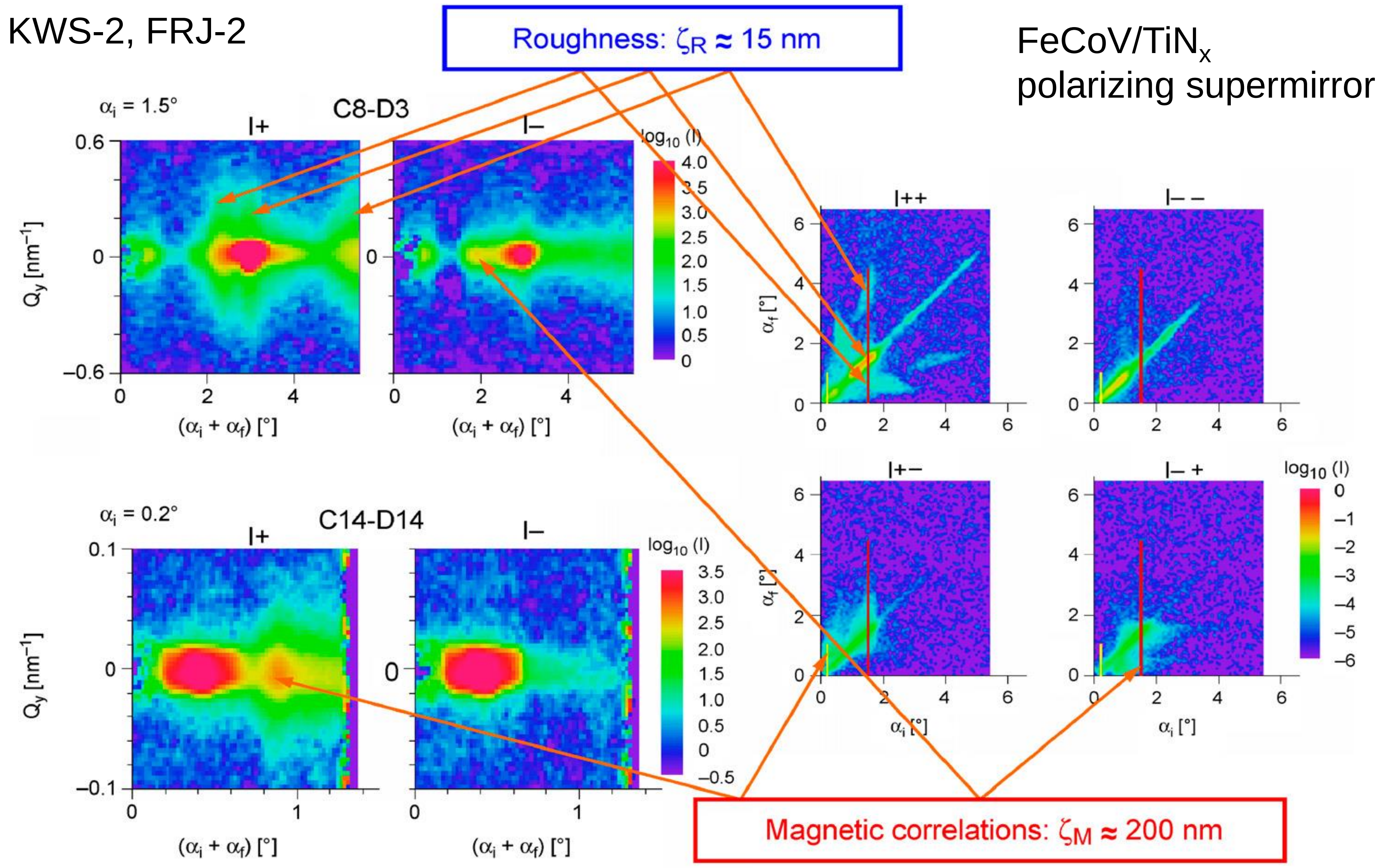
CoO [2 nm] / Co [20 nm]

HZB



H. Zabel, Materials Today 9 (2006)

GISANS. Polarized neutrons



E. Kentzinger, et al., Physica B 397 (2007) 43–46

Summary to

'Reflectometry of non-polarized and polarized neutrons from solid interfaces'

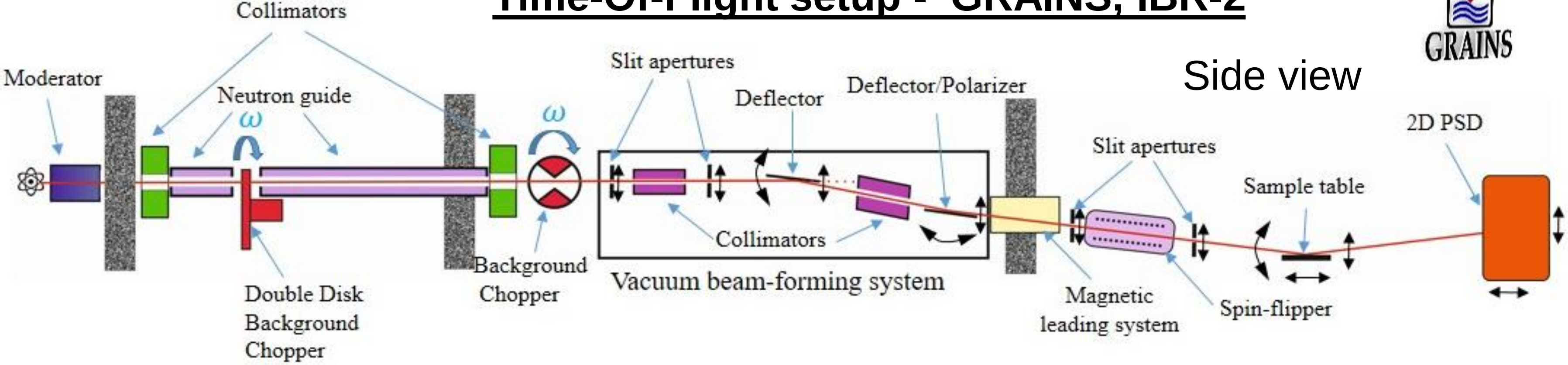
- ***Three neutron reflectometry modes are efficient in the characterization of the multilayered interface structures at interfaces. In addition to the structure, modulation along the depth profile, the characteristic of the distributions in in-plane and out-of-plane correlations are well determined using specific features in off-specular scattering and GISANS patterns.***

- ***Polarized neutron reflectometry is very efficient in the study of magnetic layered interface structures. The characteristics of the magnetic scattering length density distribution at interfaces are related to the magnetization distribution. Thus, the method represents a spatial magnetometry.***

- Principles of Neutron Reflectometry
- Reflectometry of Non-Polarized and Polarized Neutrons from Solid Interfaces
- **Neutron Reflectometry from Interfaces with Liquids**
- Experimental Aspects of Neutron Reflectometry

Horizontal reflectometers

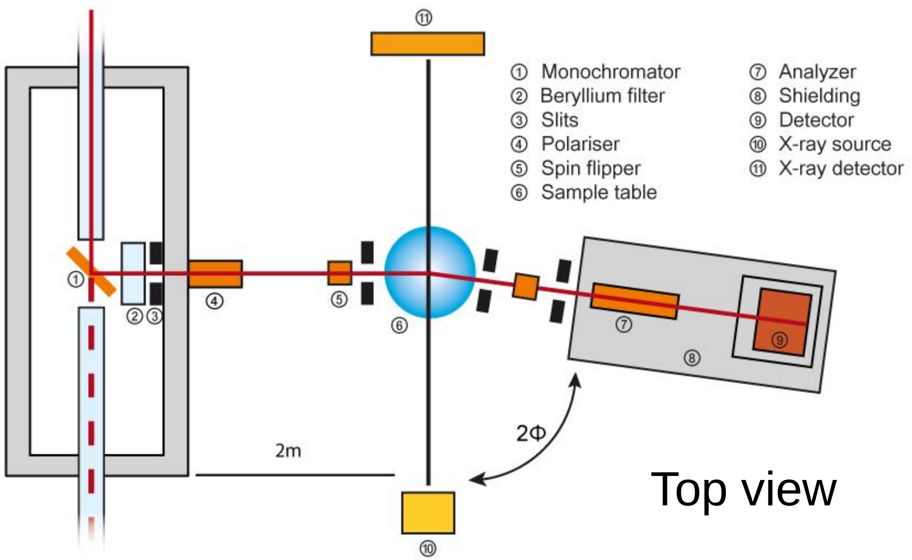
Time-Of-Flight setup - GRAINS, IBR-2



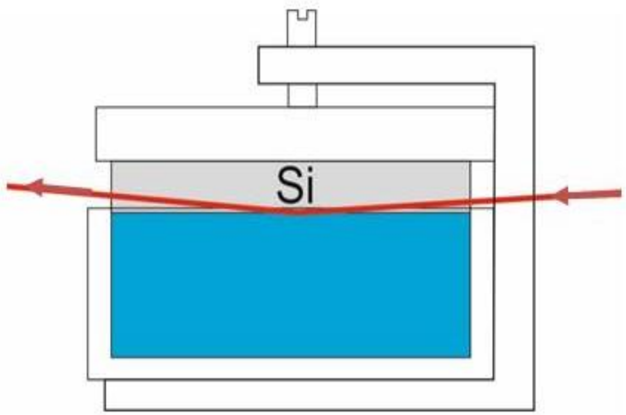
Steady-State setup - N-REX, MLZ

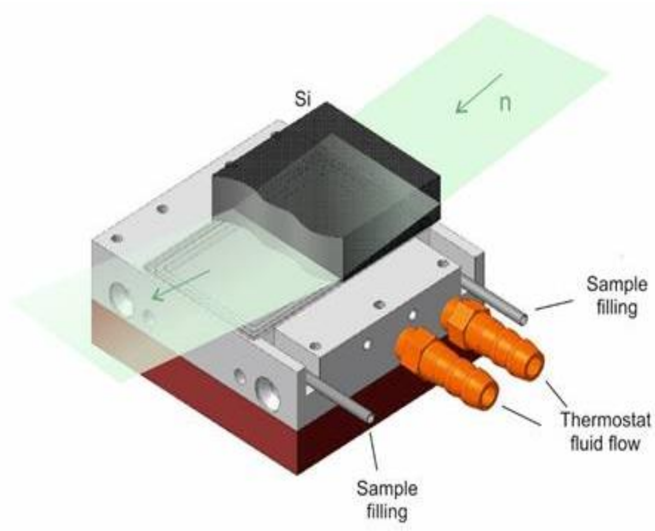
Horizontal sample plane → studies of interfaces with liquids:

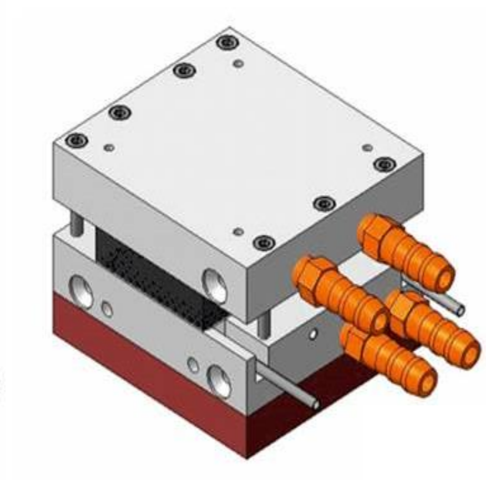
- Solid – Liquid;
- Air – Liquid;
- Liquid – Liquid



NR from Solid-Liquid interfaces

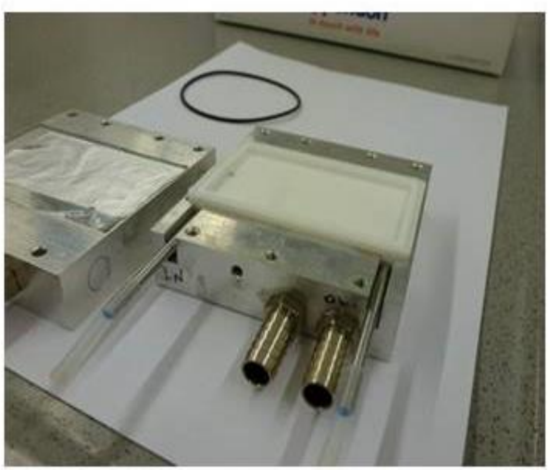


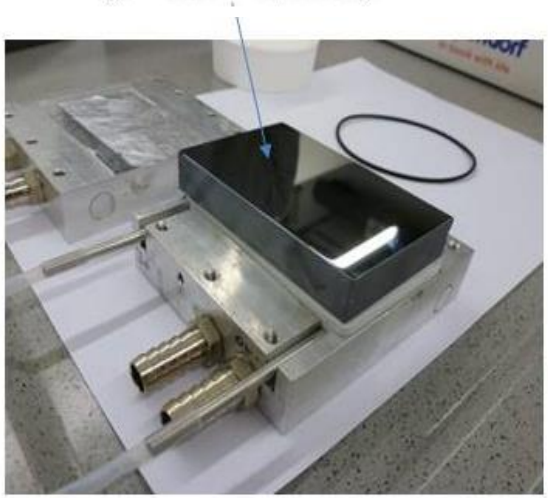


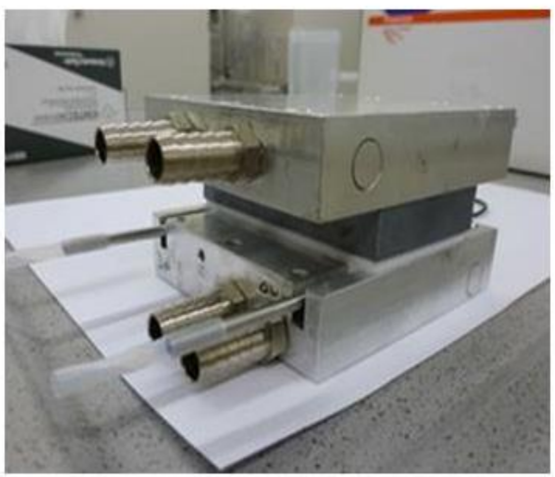


Sample volume
5-10 ml

Si crystal block
(5 × 7.5 × 1.5 cm^3)

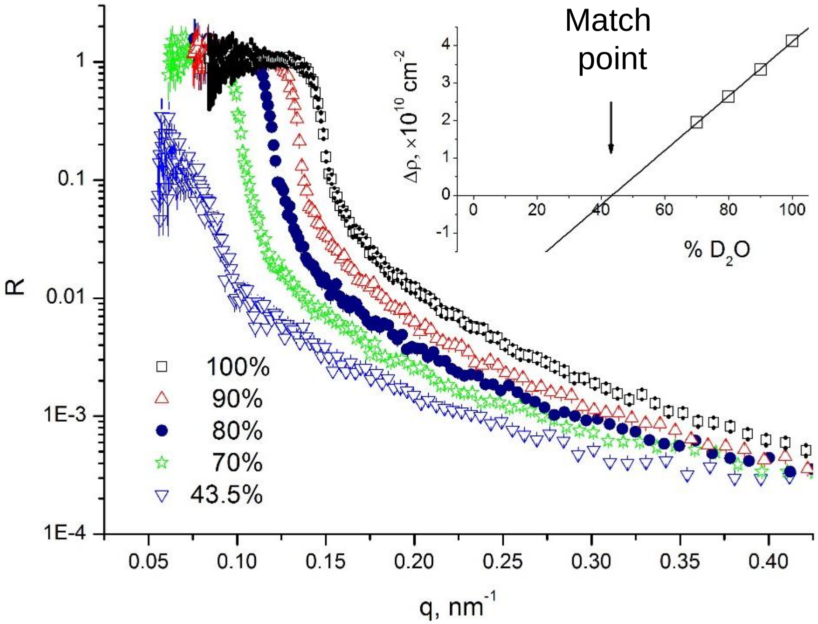




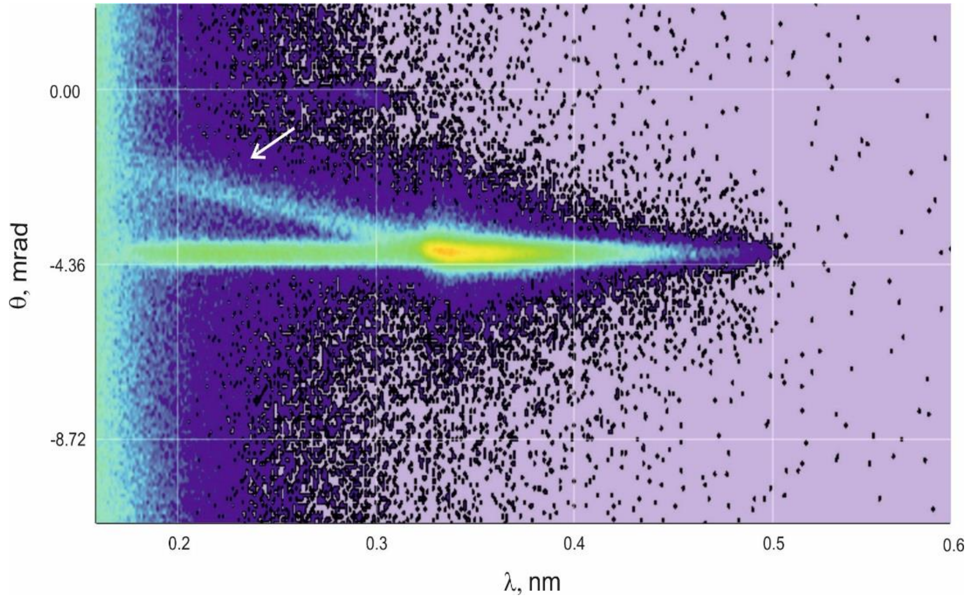


Simplest interface: Silicon - Water

Specular reflectivity: contrast variation



Diffuse scattering:
long-period non-homogeneities of substrate

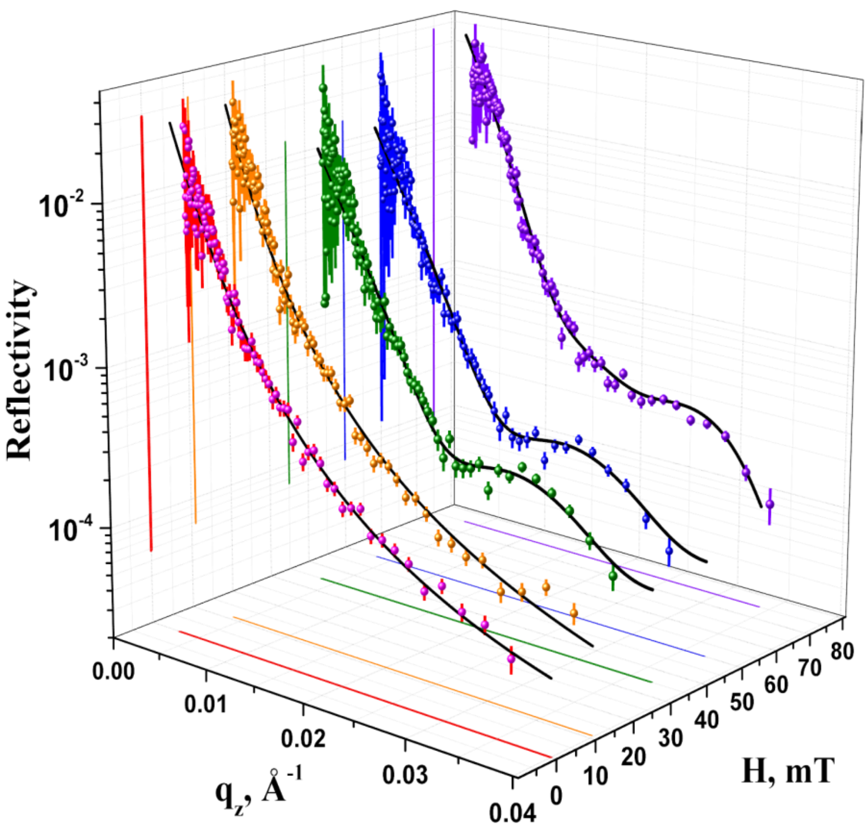


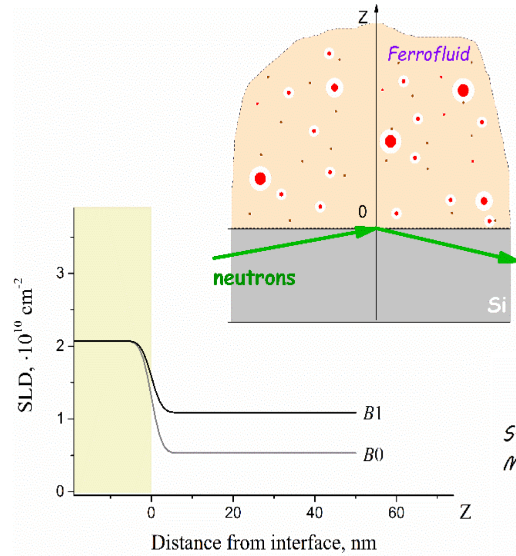
TOF mode (GRAINS, Dubna)

M.V. Avdeev, V.I. Bodnarchuk, V.I. Petrenko, I.V. Gapon, O.V. Tomchuk, A.V. Nagornyi, V.A. Ulyanov, L.A. Bulavin, V.L. Aksenov, Crystallography Reports 62 (6) (2017) 1002–1008

Interface Assembling of Magnetic Nanoparticle from Ferrofluids Induced by Non-Homogeneous Magnetic Field

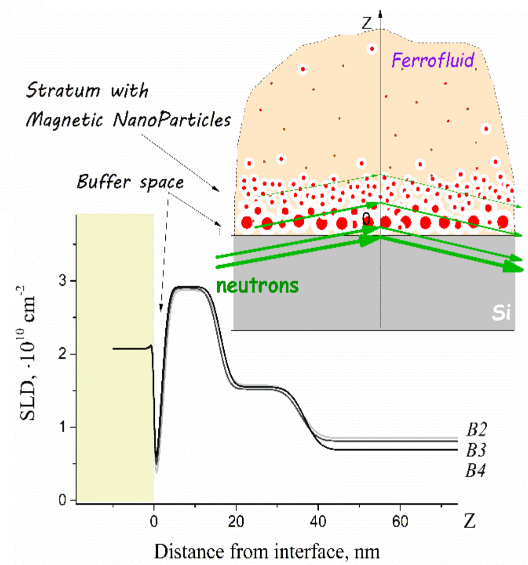
Nanomagnetite (∅ 2-200 nm) in transformer oil, φ_m ~ 6%





∇H

zero and weak magnetic fields

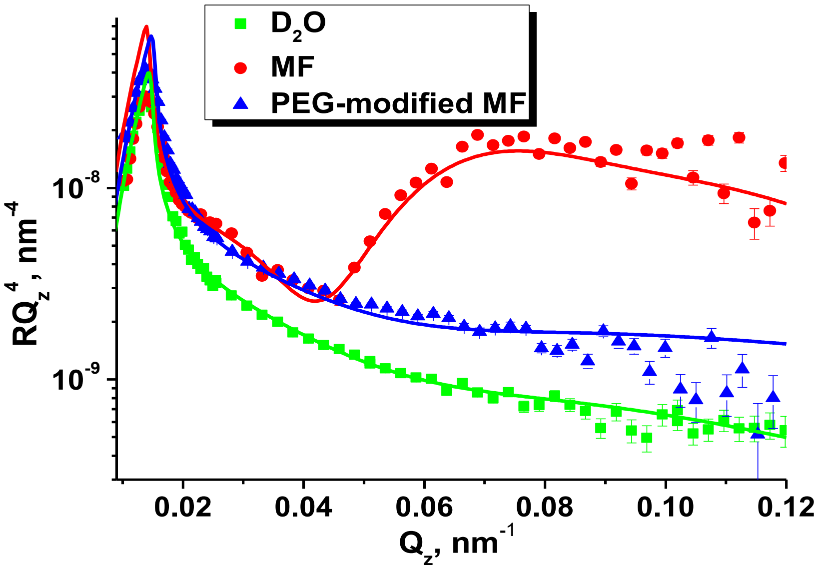


GRAINS, IBR-2, Dubna

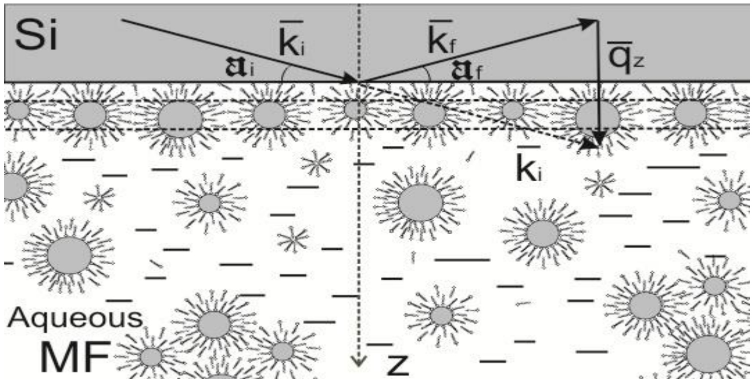
higher magnetic flux density (35-75 mT)

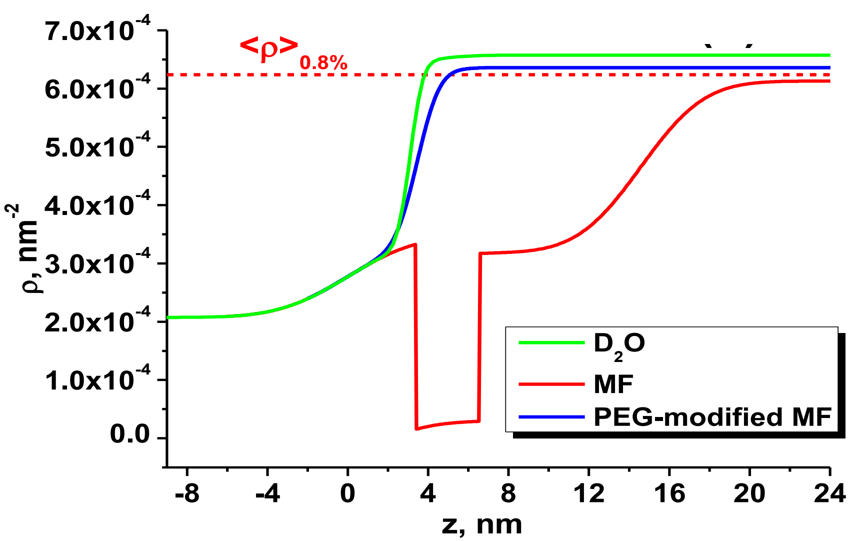
A.V.Nagornyi, V.I.Petrenko, M.Rajnak, I.V.Gapon, M.V.Avdeev, et al. *Appl. Surf. Sci.* 473 (2019) 912–917.

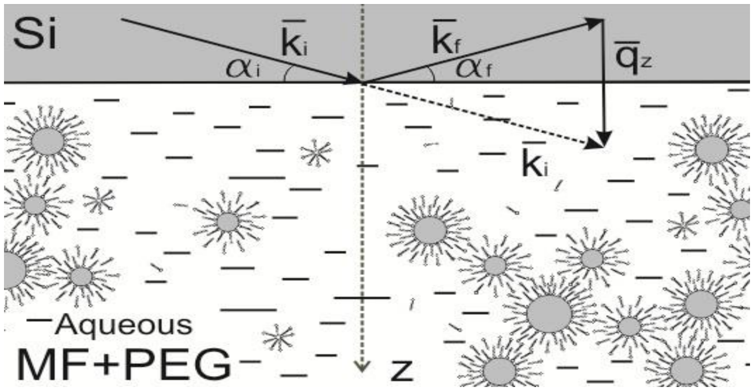
Adsorption of nanoparticles
from aqueous magnetic fluids on silicon substrate



Effect of polymer (PEG) modification







PEG – poly(ethylene glycol)

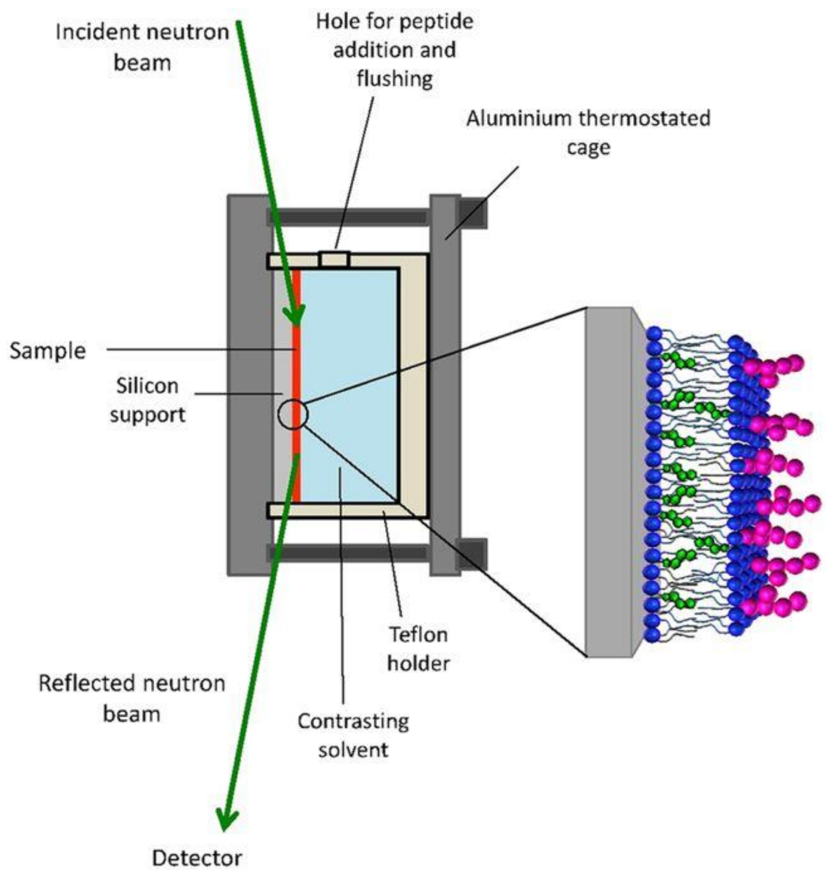
NREX+, FRM-II, Munich

M.V. Avdeev, V.I. Petrenko, I.V. Gapon, L.A. Bulavin, A.A. Vorobiev, O. Soltwedel, M. Balasoiu, L. Vekas, V. Zavisova, P. Kopcansky, *Appl. Surf. Sci.* 352 (2015) 49

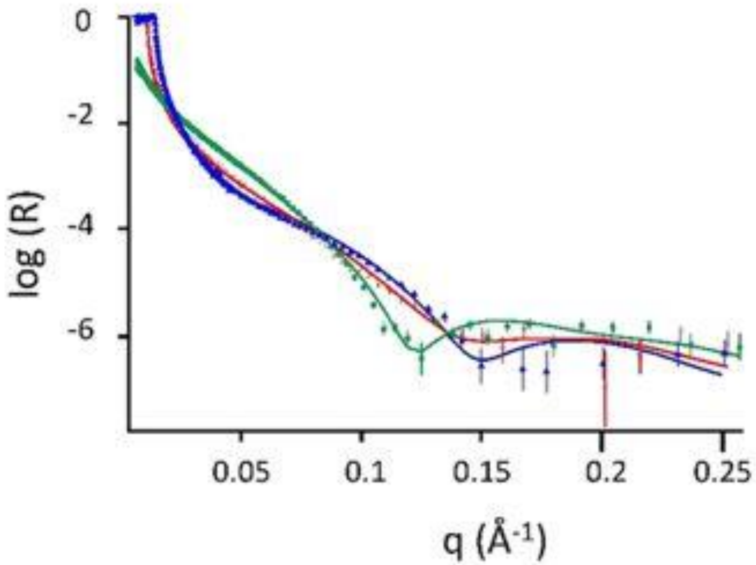
M.Kubovcikova, I.V.Gapon, V.Zavisova, M.Koneracka, V.I.Petrenko, O.Soltwedel, L.Almasy, M.V. Avdeev, P.Kopcansky, *J. Magn. Magn. Mater.* 427 (2017) 67

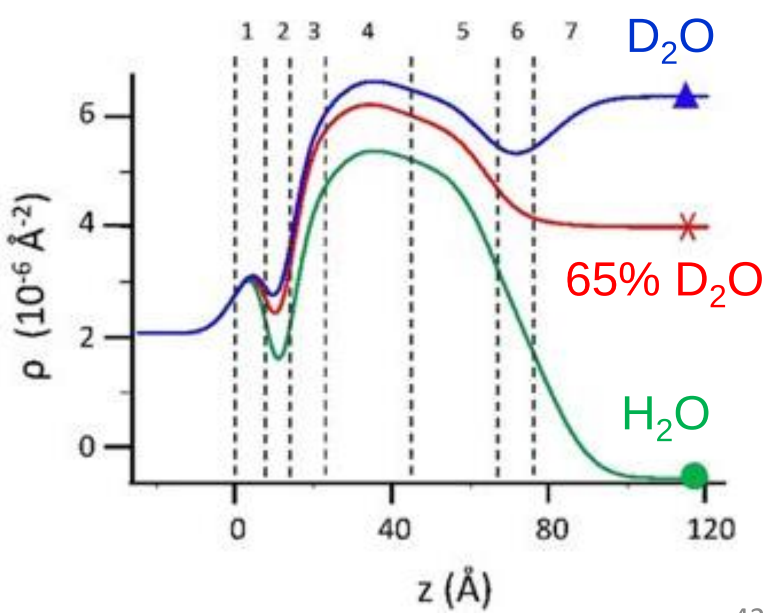
Amyloidβ Peptides in interaction with raft-mime model membranes

Vertical geometry



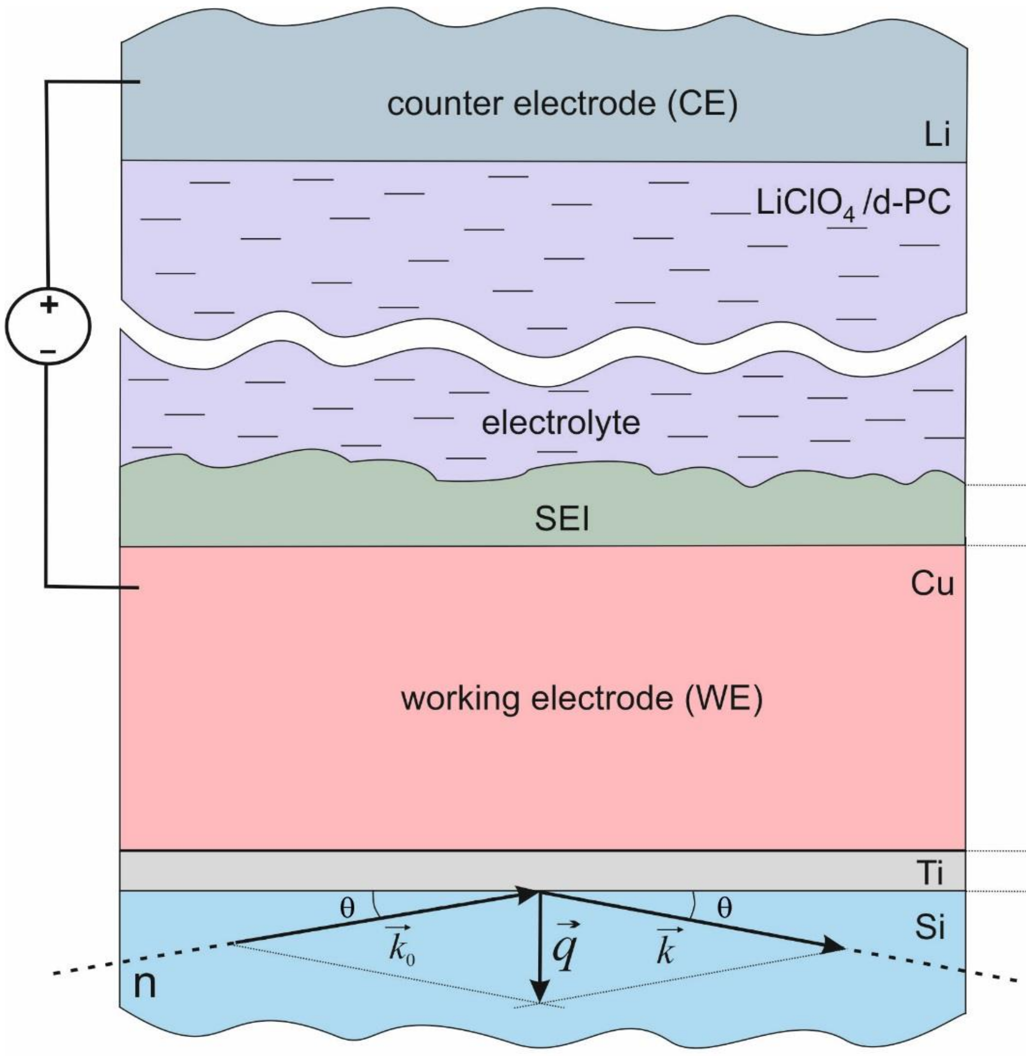
D17, ILL, Grenoble



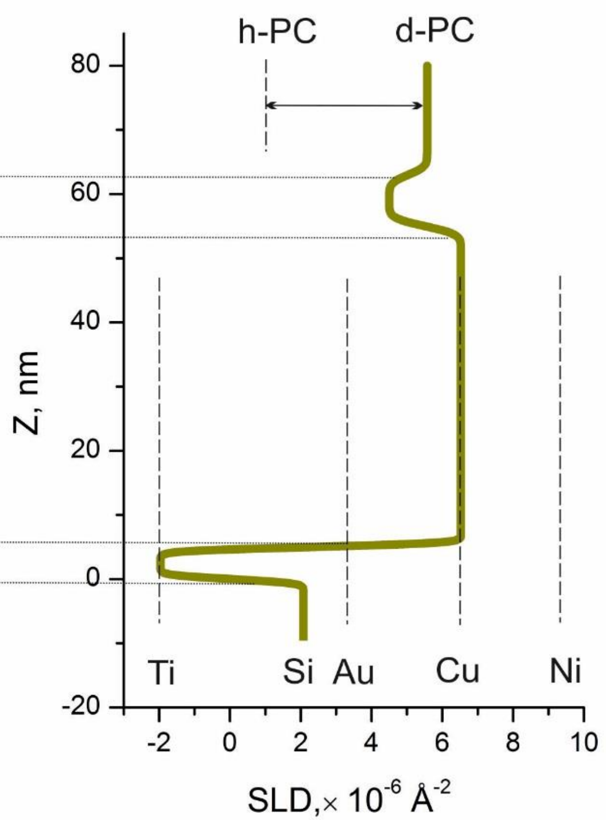


V. Rondelli, P. Brocca, S. Motta, et al., *Scientific Reports* 6, 20997 (2016)

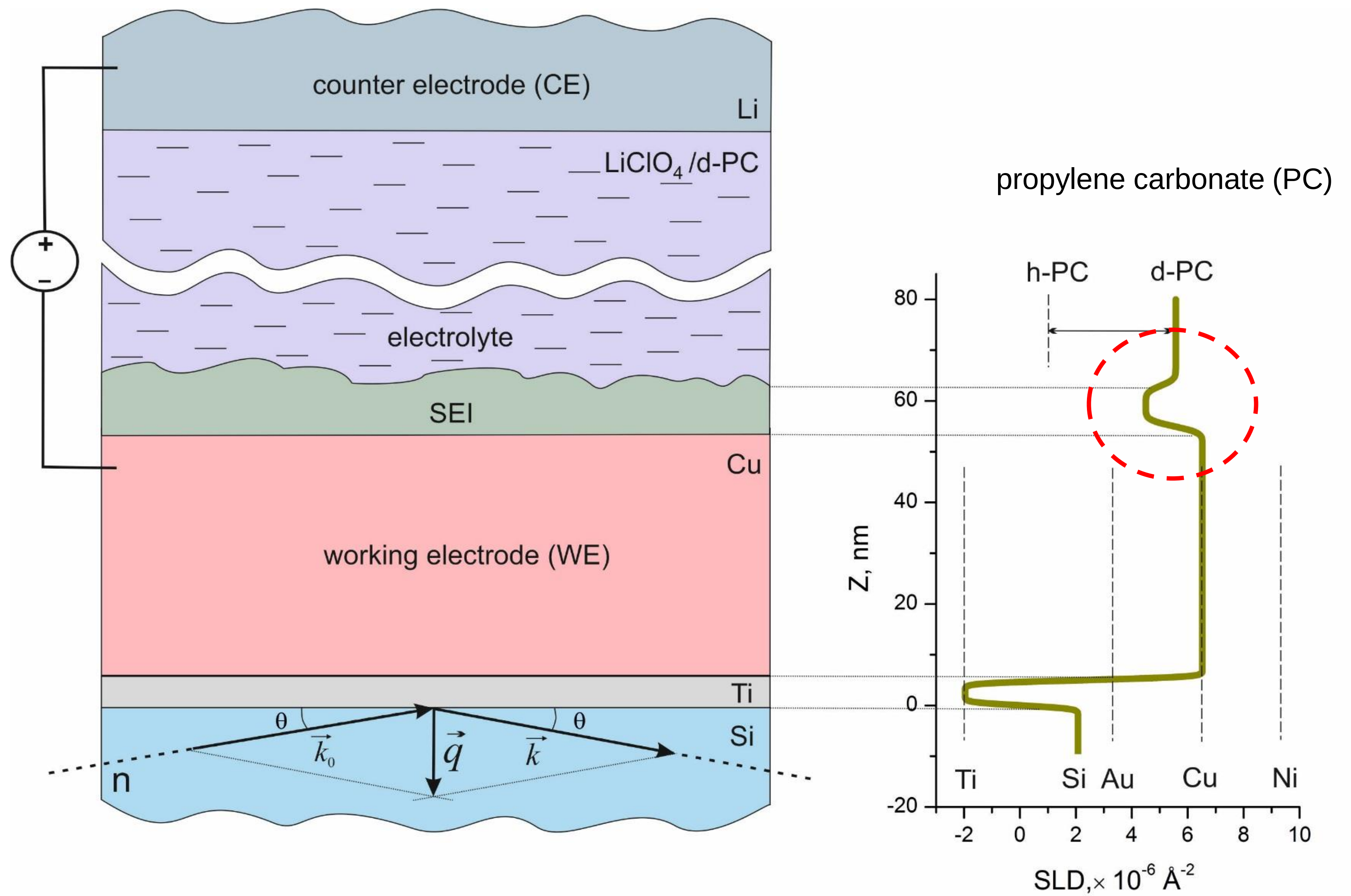
Study of Electrolyte-Electrode Interfaces by NR



propylene carbonate (PC)

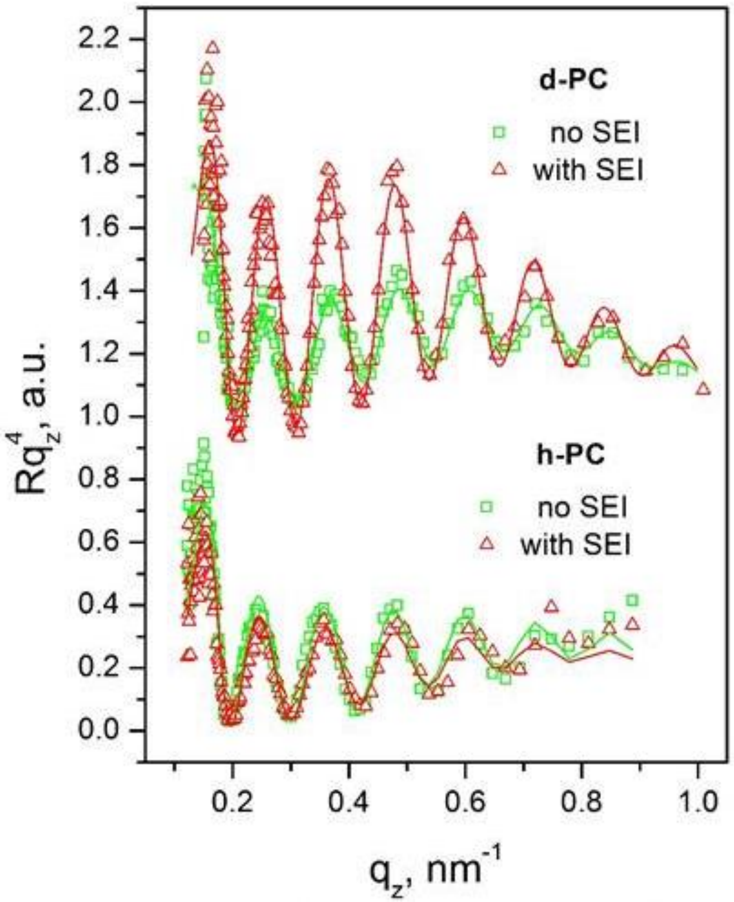


Study of Electrolyte-Electrode Interfaces by NR

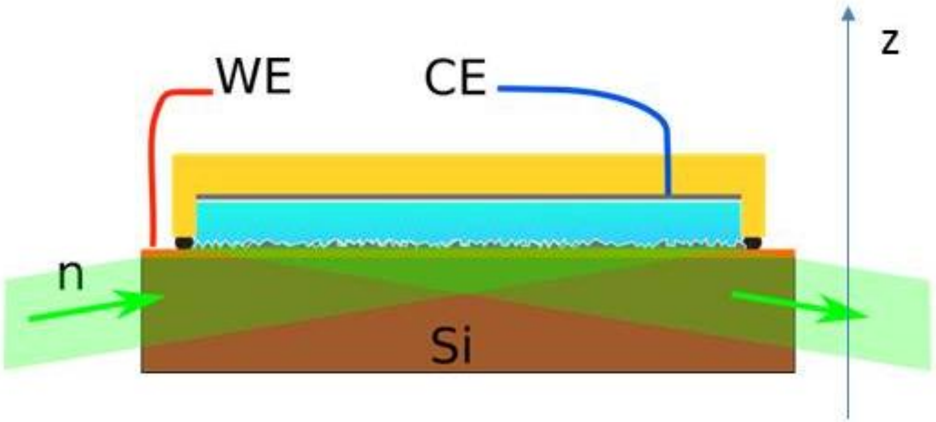


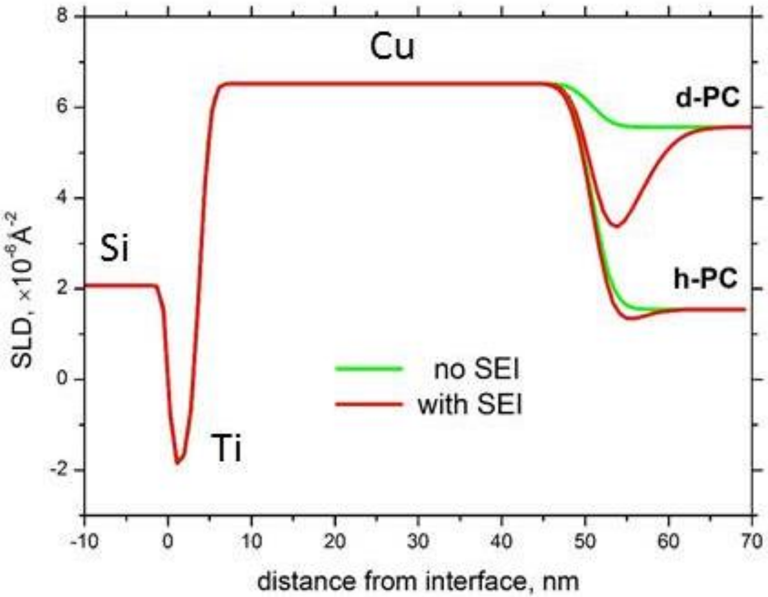
Study of Electrolyte-Electrode Interfaces by NR

Formation of Solid Electrolyte Interphase (SEI) on plane metal anodes



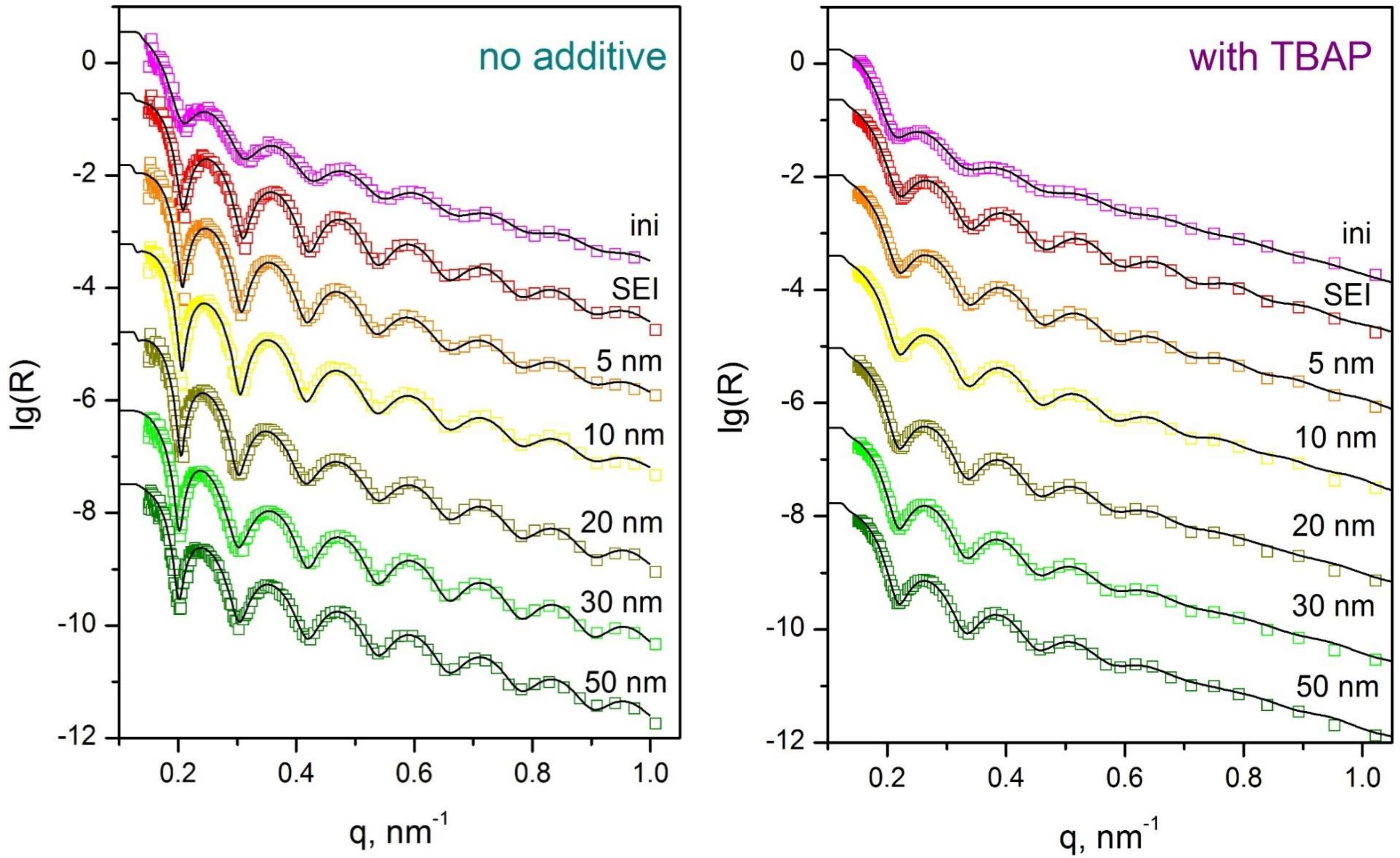
GRAINS TOF Reflectometer, IBR-2, Dubna





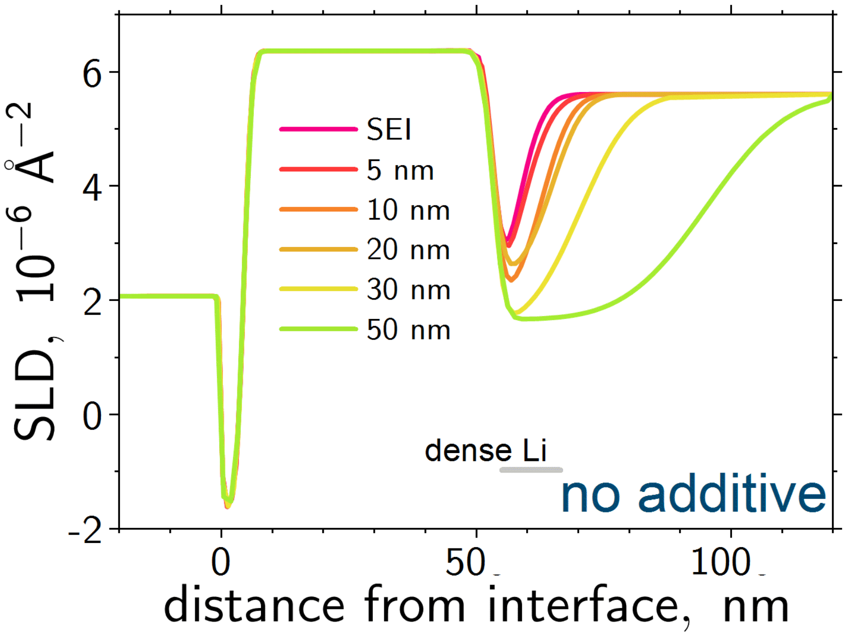
Effect of TBAP: neutron reflectometry

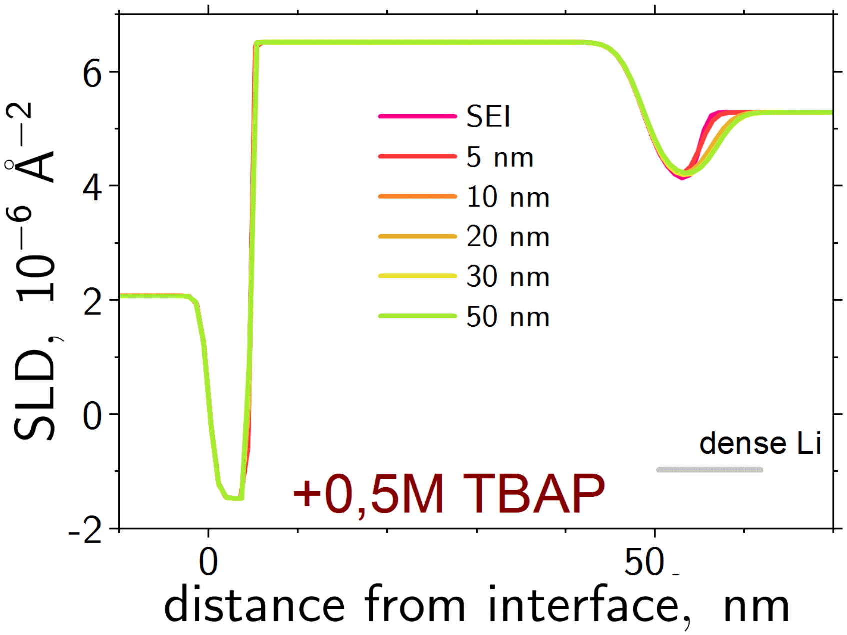
tetra-n-butylammonium perchlorate (TBAP)

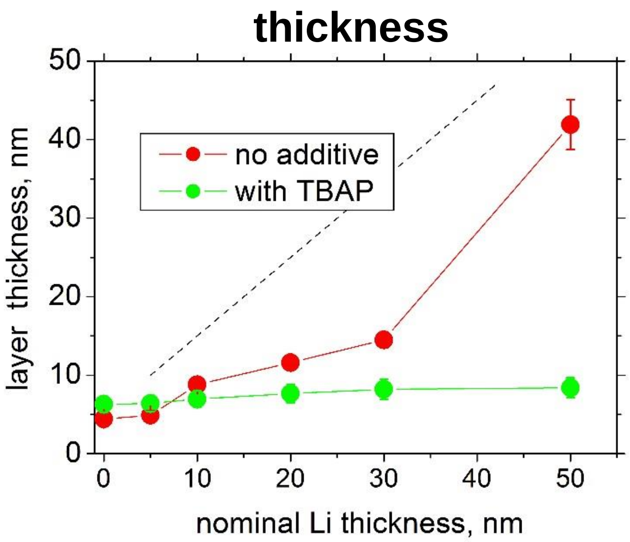


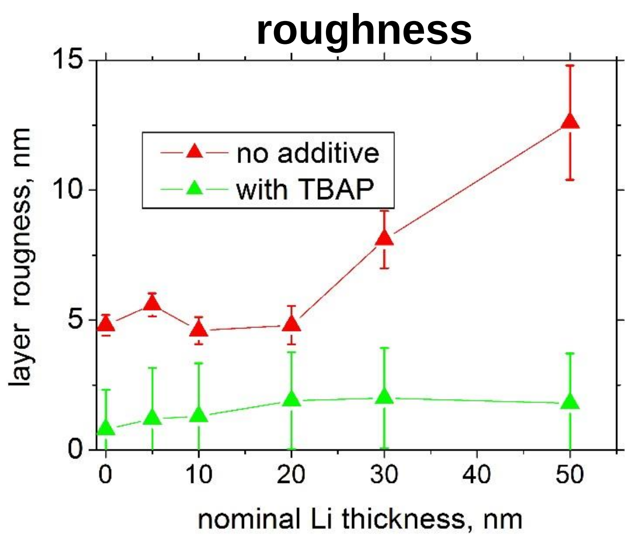
M.V.Avdeev, Rulev A.A., Bodnarchuk V.I., et al. Appl. Surf. Sci, 424 (2017) 378
M.V. Avdeev, A.A. Rulev, E.E. Ushakova, et al., Appl. Surf. Sci. 486 (2019) 287

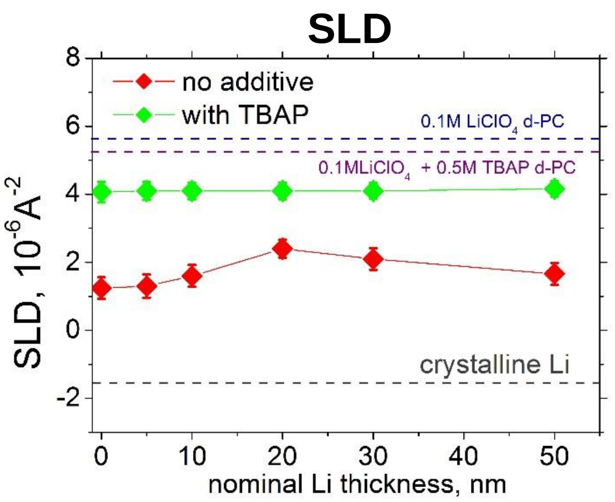
Effect of TBAP: neutron reflectometry



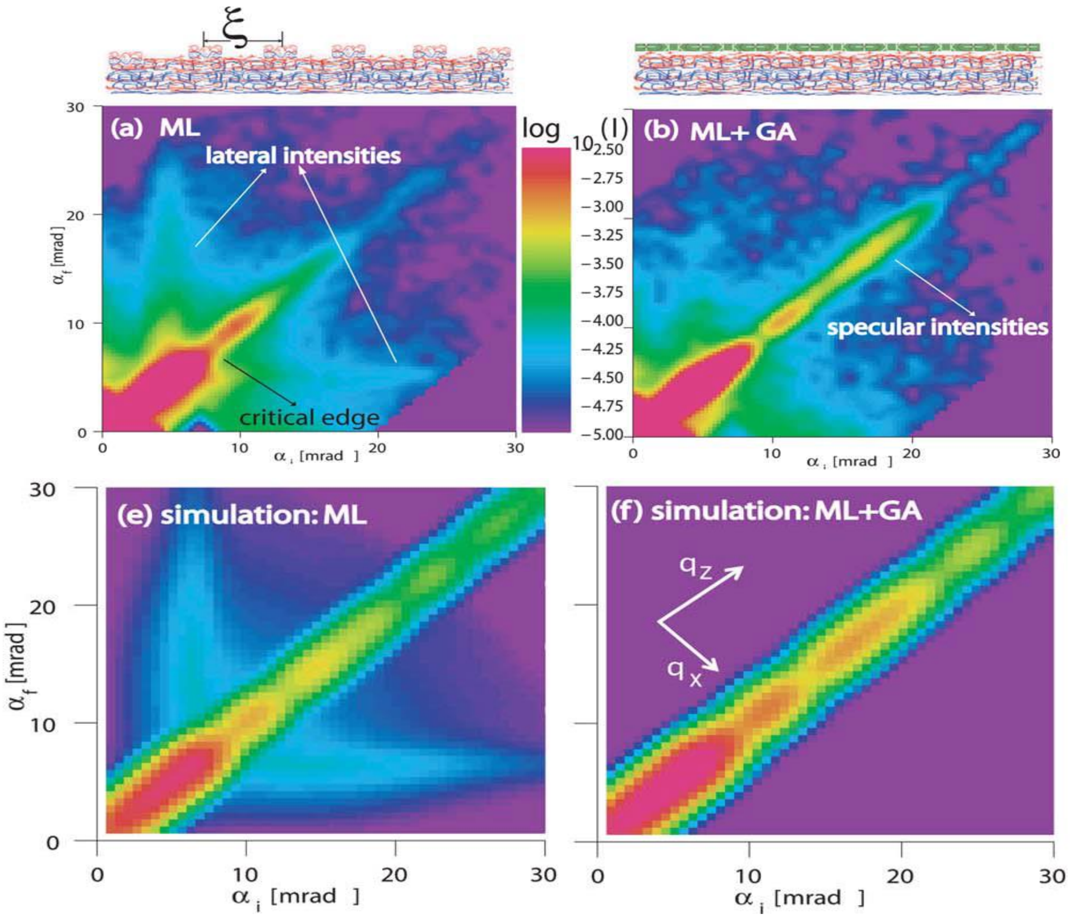








Off-specular mode. Solid-liquid interfaces



Smoothening of polyelectrolyte multilayers with molecular additive (glutamic acid)

Adsorption of polyelectrolytes on Si from D_2O

sample ML: $PEI/((PSS/PAH)_2/(dPSS/PAH))_3$

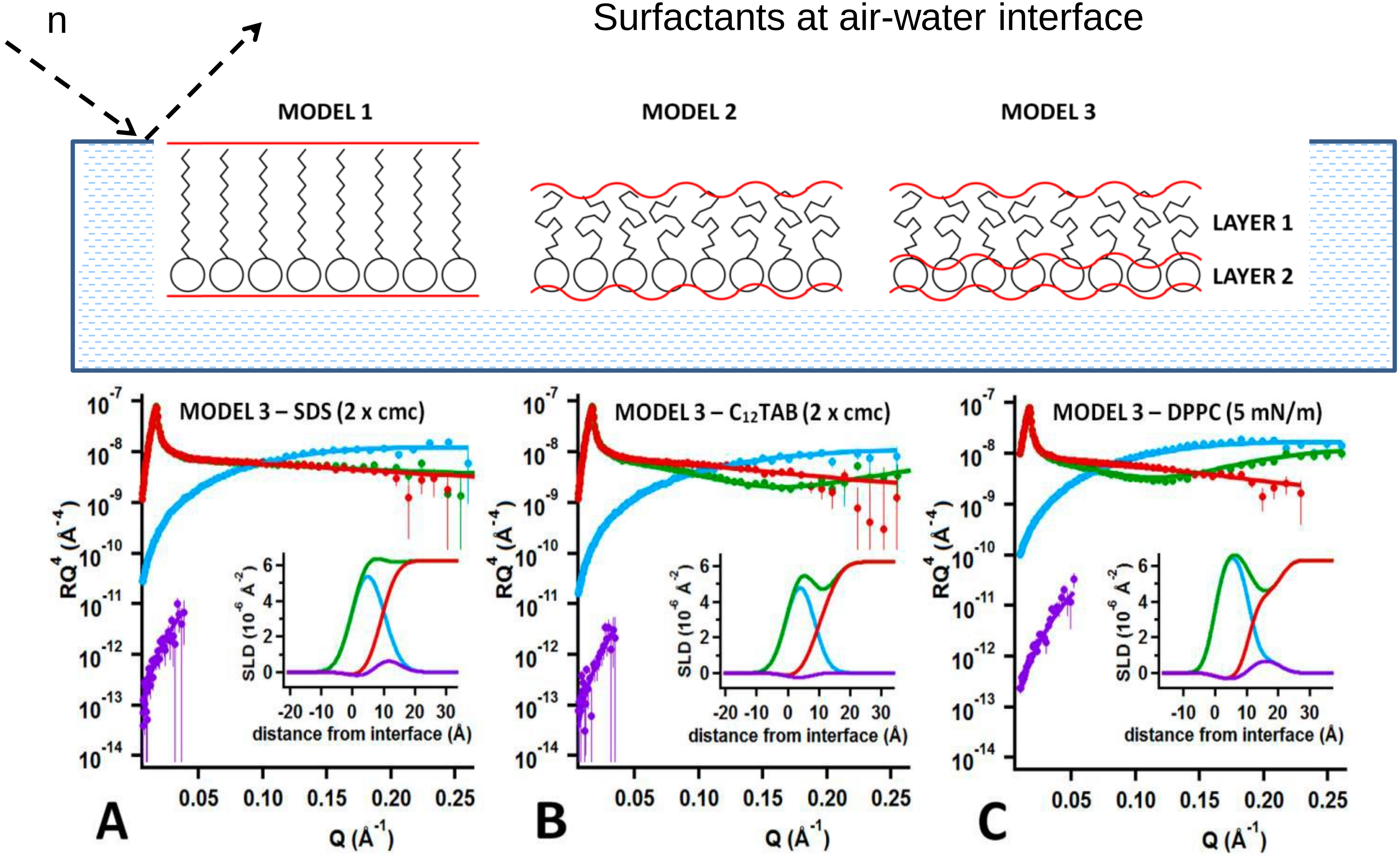
sample ML + GA: $PEI/((PSS/PAH)_2/(dPSS/PAH/GA))_3$

TREFF, MLZ

N. Paul, A. Paul, S. Mattauch, et al., Soft Matter 9 (2013) 10117

Neutron reflectometry from free surfaces

Surfactants at air-water interface



FIGARO, ILL

R.A. Campbell, et al., J. Coll. Interface Sci. 531 (2018) 98-108.

Summary to

‘Reflectometry from Interfaces with Liquids’

- ***Neutron reflectometry experiments for interfaces with liquids require special cells and sample environment. The active development of such kind of experiments for soft matter research is due to high penetrating power of neutrons and possibilities for enhancing reflectivity by varying the contrast (using deutaration).***

- ***The behavior of colloidal liquid solutions at interfaces with solids is an important area of research with the practical impact.***

- ***Off specular scattering is sensitive to fine structural effects in liquid-solid interfaces with colloidal solutions.***

- ***Study of structural organization of free surfaces of air-liquid interfaces is of current interest.***

- Principles of Neutron Reflectometry
- Reflectometry of Non-Polarized and Polarized Neutrons from Solid Interfaces
- Neutron Reflectometry from Interfaces with Liquids
- Experimental Aspects of Neutron Reflectometry

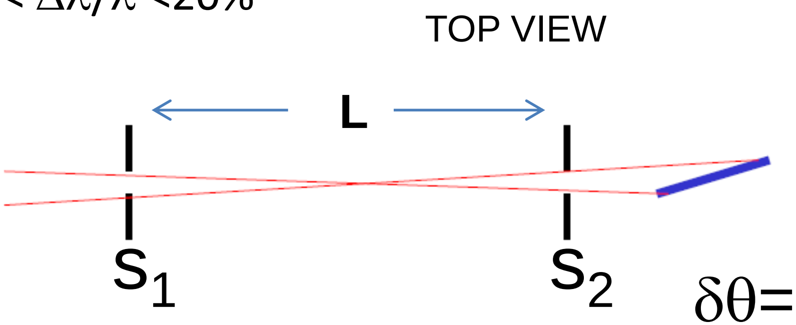
Resolution function: vertical sample geometry

Wavelength resolution:

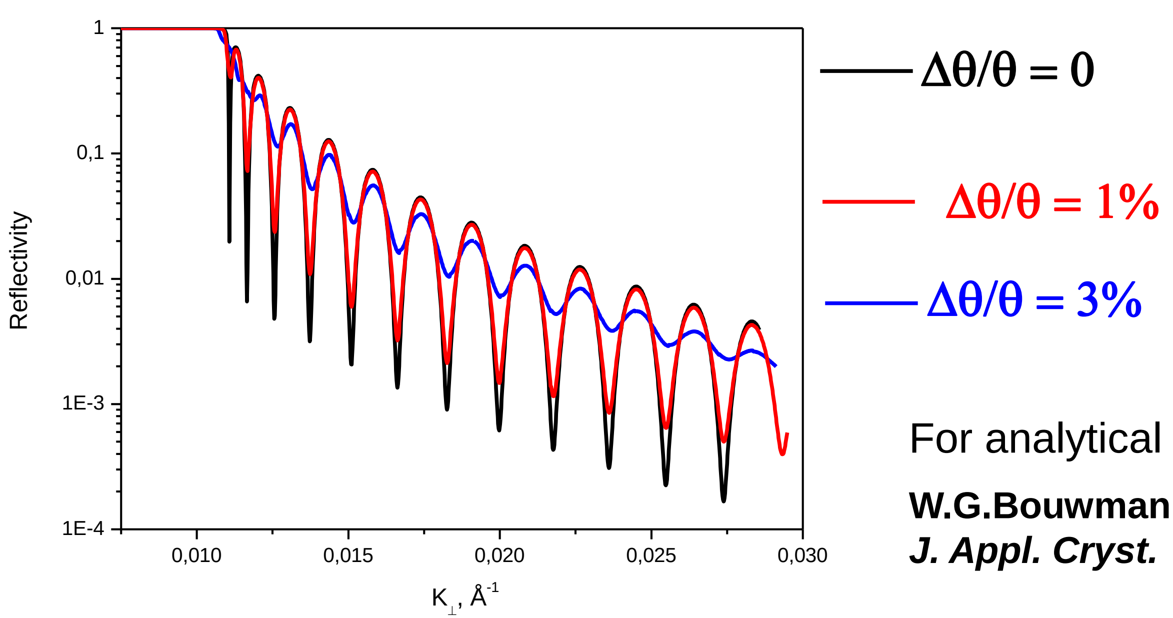
- Graphite monochromator $\Delta\lambda/\lambda \sim 1\%$
- Multilayered monochromator $5\% < \Delta\lambda/\lambda < 20\%$
- TOF-mode $1\% < \Delta\lambda/\lambda < 20\%$

Angular resolution:

- Determined by aperture sizes



$$\delta\theta = (s_1 + s_2)/L$$

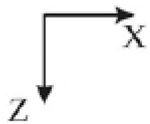


For analytical representation see

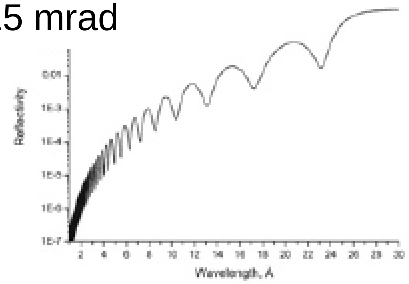
W.G.Bouwman, J.S.Pedersen,
***J. Appl. Cryst.* 29 (1996) 152-158**

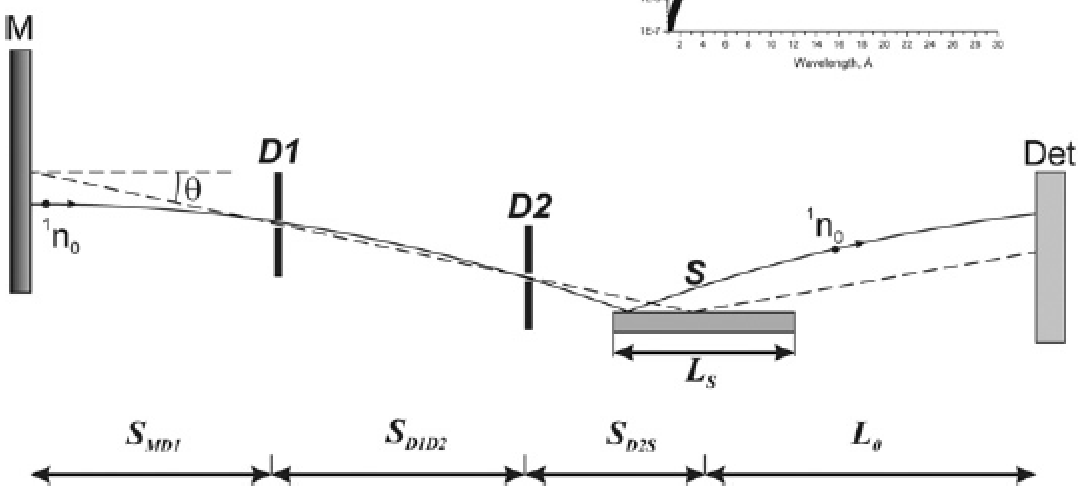
Resolution function: horizontal sample geometry

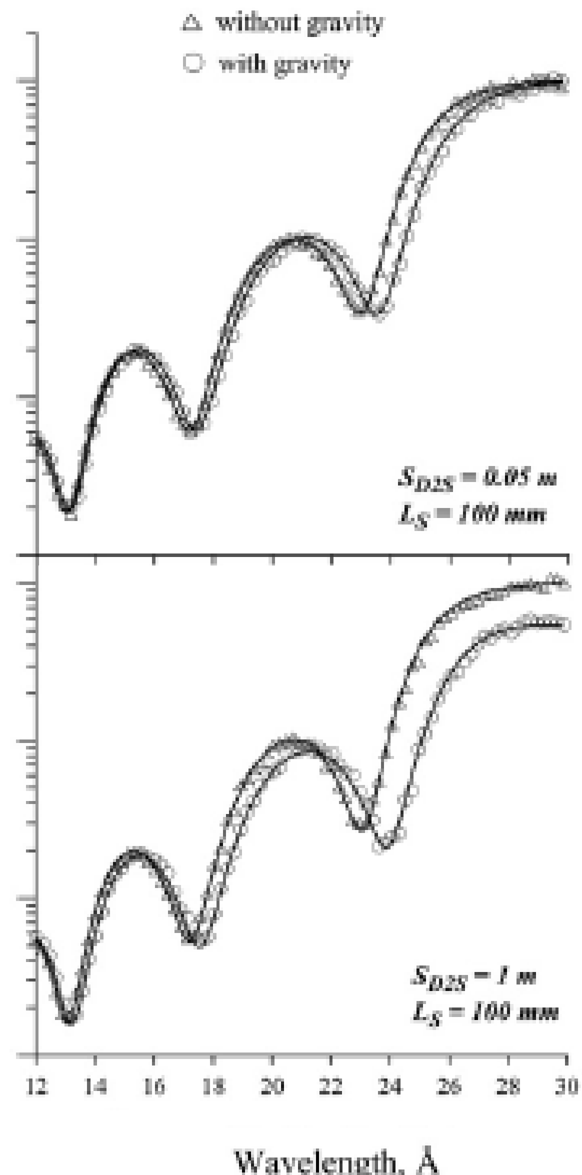
Additional gravity effect is important for λ > 1 nm



Model monolayer
d = 150 nm ; θ = 15 mrad



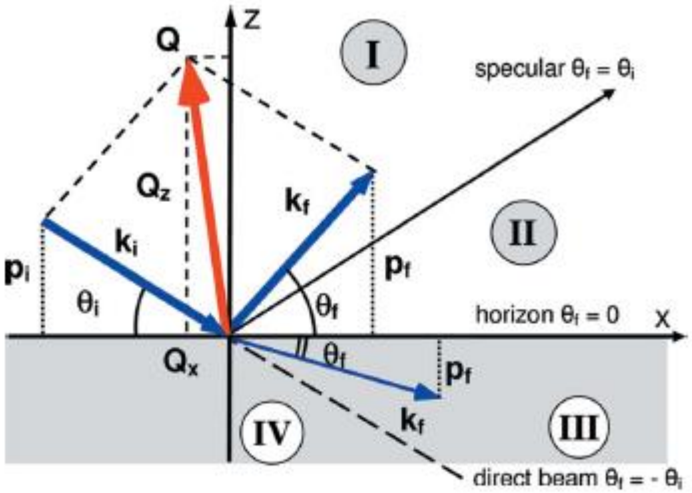




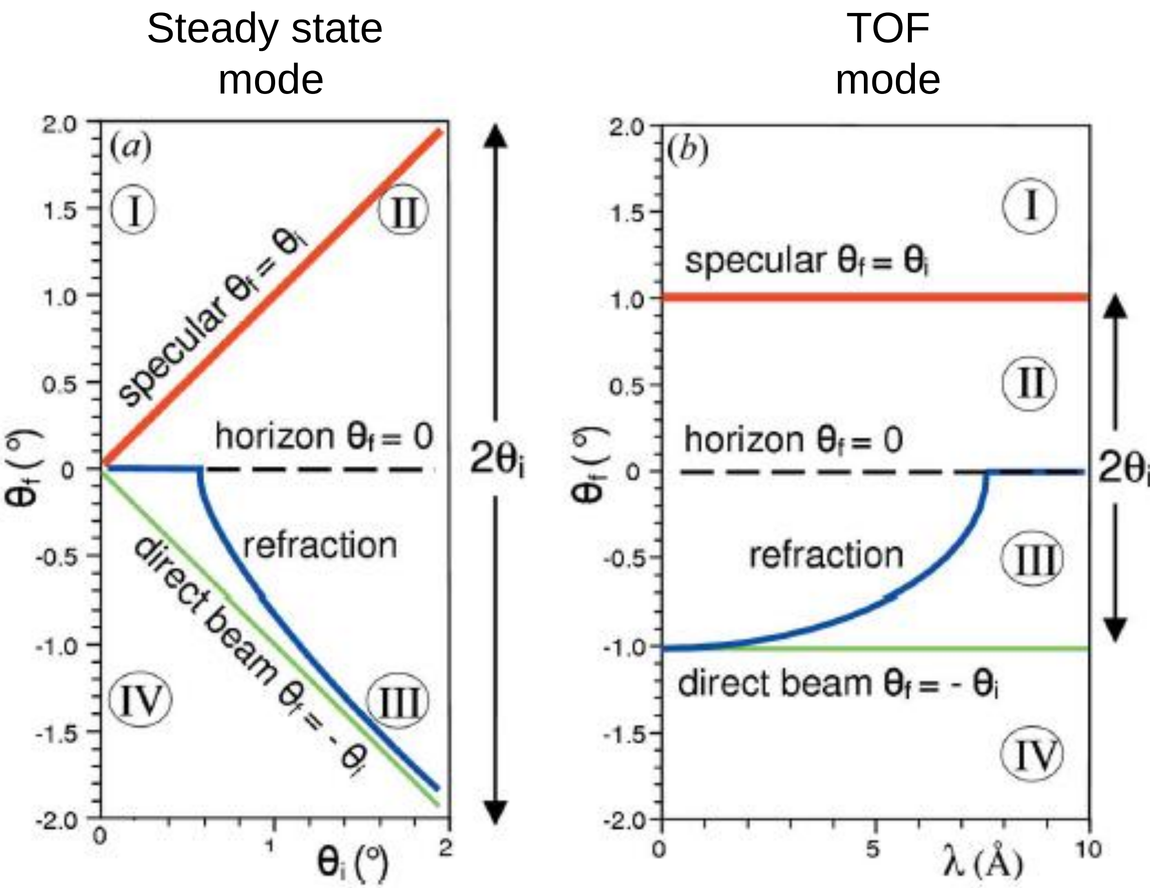
For analytical representation see

I.Bodnarchuk, et al.,
Nucl. Instr. Methods A 631 (2011) 121–124

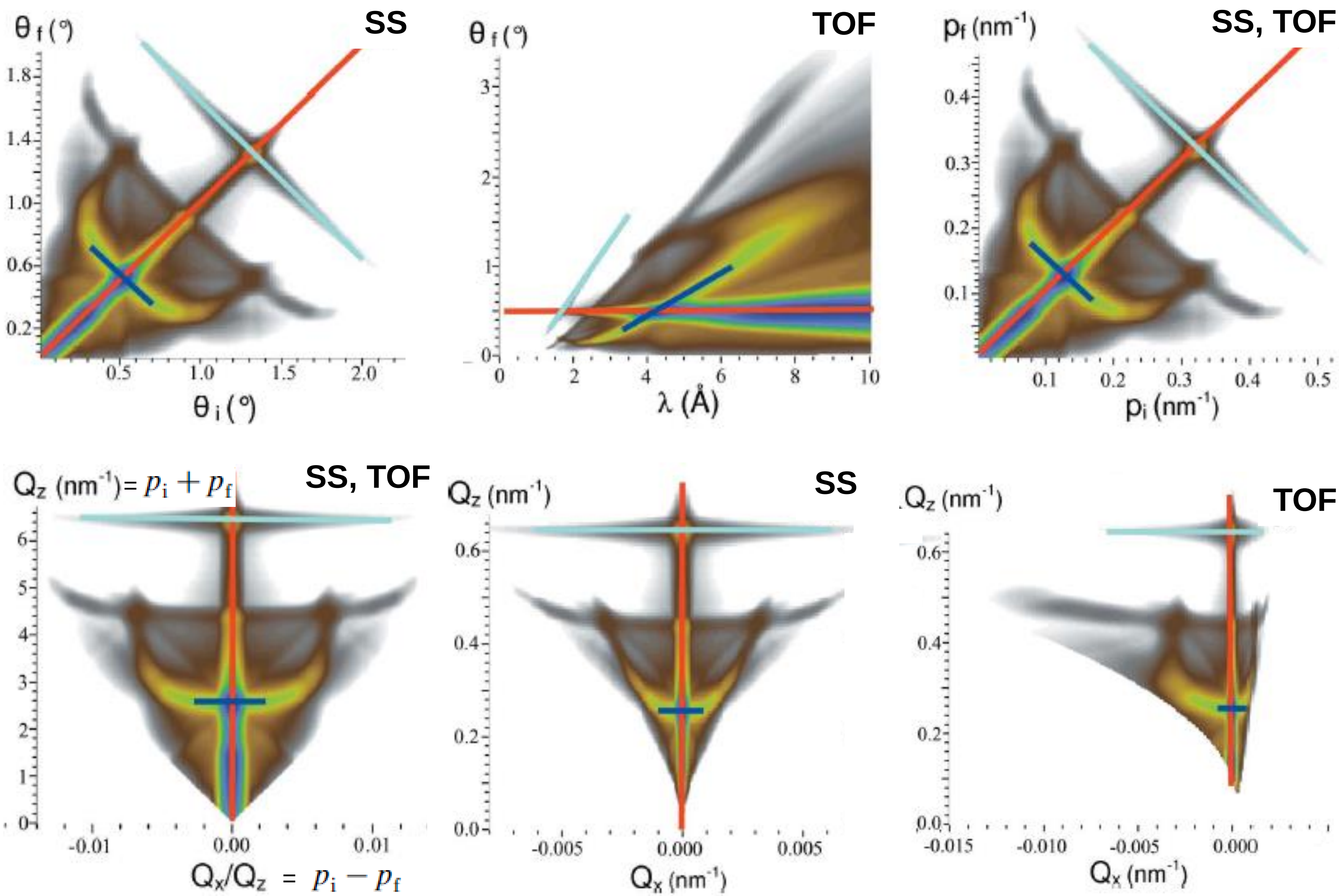
Representation of NR data: specular scattering



$$Q_x = (\pi/\lambda)(\theta_i + \theta_f)(\theta_i - \theta_f)$$
$$Q_z = (2\pi/\lambda)(\theta_i + \theta_f).$$



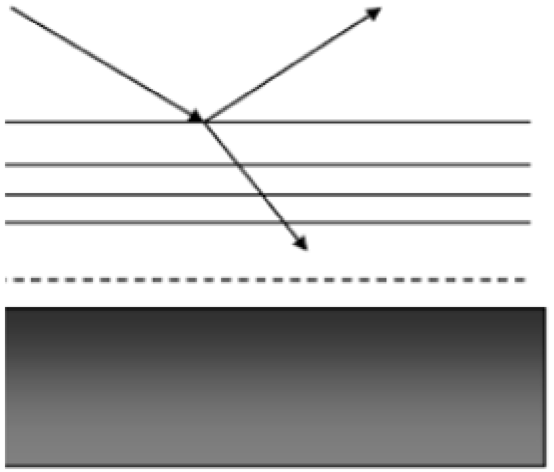
Representation of NR data: diffuse scattering



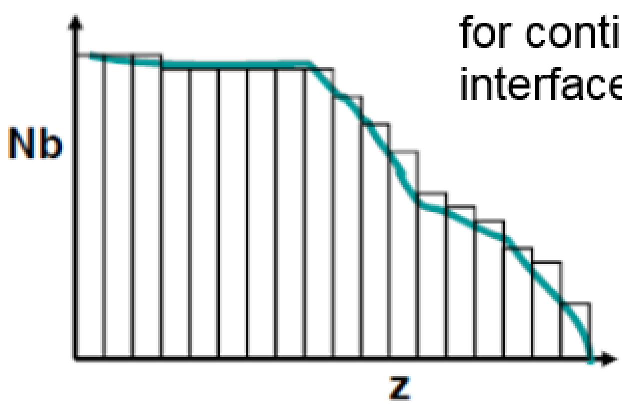
For details see **F.Ott, S.Kozhevnikov J. Appl. Cryst. 44 (2011) 359–369**

Analysis of NR data: matrix formalism for specular reflectivity

for multilayers



(Born & Wolf, 'Principles in Optics', 6th Ed, Pergammon, Oxford, 1980)



for continuous interfaces

Parrat formalism

For *j*-th layer

$$M_j = \begin{bmatrix} \cos\beta_j & -(i/p_j)\sin\beta_j \\ -ip_j\sin\beta_j & \cos\beta_j \end{bmatrix}$$

$$p_j = n_j \sin\theta_j \qquad \beta_j = (2\pi/\lambda) n_j d_j \sin\theta_j$$

Total reflectivity matrix

$$M = [M_1].[M_2]----[M_n]$$

Reflectivity

$$R^2 = \left| \frac{(M_{11} + M_{12} p_s) p_0 - (M_{21} + M_{22}) p_s}{(M_{11} + M_{12} p_s) p_0 + (M_{21} + M_{22}) p_s} \right|^2$$

Software: Parrat32, Motofit / IGOR PRO, EFFI, SansView, MAUD …

Summary to

'Experimental Aspects of Neutron Reflectometry'

- ***Resolution function of neutron reflectometer contains wavelength and angular components which in optimum should be close. For reflectometers with horizontal sample plane gravitational effect for long-wave neutrons should be taken into account;***

- ***Off-specular scattering patterns can be represented in different systems of coordinates depending on the mode of experiment and specific scattering features;***

- ***Matrix formalism is one of the mostly used approaches for calculating specular reflectivity and fitting it to experimental curves.***